Theory and Analysis of Quartz Crystal Resonators

A Tutorial at the 2010 IEEE International Frequency Control Symposium

by

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Outline

1. History and trends of quartz crystal resonators
2. Fundamentals of wave propagation
3. Quartz crystal material
4. Thickness vibrations of infinite plates
5. Mindlin plate equations
6. Complication factors
7. Analytical considerations
8. Finite element method

Part II
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Analytical Considerations

1. 3-D piezoelectric equations for quartz with material dissipation
2. Butterworth Van Dyke resonator model of the quartz resonator: the equivalent electrical parameters
3. $Q$ of a resonator
4. Eigenvalue analysis
5. Frequency response analysis
3-D piezoelectric equations for quartz \textit{with} material dissipation

- Strain-displacement: $S_{ij} = \frac{1}{2} (U_{j,i} + U_{i,j})$
- Constitutive Equation: $T_{ij} = (C_{ijkl} + j\omega\eta_{ijkl})S_{kl} + e_{kij}\phi_k$
- Electrostatic Constitutive Equation: $D_i = e_{ijk}S_{jk} - \varepsilon_{ik}\phi_k$
- Stress equation of motion: $T_{ij,j} = -\omega^2 \rho U_i \text{ in } V$
  Specify $p_i = n_j T_{ij}$ or $U_i \text{ in } S$
- Charge equation of motion: $D_{i,i} = 0 \text{ in } V$
  Specify $q = n_i D_i$ or $\phi \text{ in } S$
Butterworth Van Dyke resonator model of the quartz resonator

Harmonic excitation at frequency $\omega$

\[ X_1 = \omega L_1 - 1/(\omega C_1) \]

\[ X_0 = -1/(\omega C_0) \]

The inductance $L_1$ and reciprocal of motional capacitance $1/C_1$ represent respectively the effective mass and stiffness of the quartz resonator, while the resistance $R_1$ is the resonator damping.

The resonator at rest is the shunt capacitor $C_0$.

Motional arm of the resonator

There are demands for frequency devices with a low phase noise and high frequency stability based on a high $Q$ factor. The devices are usually derived from the quartz crystal resonator.

Rapid miniaturization of the quartz resonator causes a lowering of the $Q$. 
Miniaturization trend of resonators
Good models are needed to reduce the cycle times for the design and prototyping of miniaturized resonators.

Accurate models for arriving at optimized designs of miniaturized quartz resonators along with an estimation of their $Q$ and equivalent circuit parameters would be very useful.

The miniaturized resonator is more sensitive to the effects of the packaging. Hence, 3-D models that include the effects of packaging are needed.
Possible factors that determine $Q$ or $R_1$ of a resonator

1. Energy loss by mechanical viscosity
2. Friction loss at the interface between the electrode film and quartz plate
3. Electrical resistance along the lead pattern
4. Mechanical vibration loss through the supporting pads
5. Friction loss via the ambient gas
6. Coupling of the main mode with spurious modes
Eigenvalue analysis of quartz resonators with material dissipation and mounting substrate
Effects of an energy sink

2-D model of a high frequency mesa plate AT-cut quartz resonator mounted on a base

- Mesa plate resonator vibrating at fundamental thickness shear mode
- Substrate upon which the mesa plate resonator was mounted
Comparison of frequency spectra using the 2-D model of the mesa plate AT-cut quartz resonator

The frequency spectrum of the mesa plate resonator mounted on a base substrate is much richer, hence there will be more interactions with spurious modes that lead to lower $Q$ factors.

Mesa plate resonator itself

Mesa plate resonator mounted on the base
Dimensions of the AT-cut quartz resonator

Blank: 4000 microns (X1) x 1500 microns (X3) x 165.47 microns (X2)
Gold Electrodes: 3000 microns (X1) x 1000 microns (X3) x 0.3 microns (X2)
10 MHz AT-cut quartz resonator with dissipation for a/b = 24-25
Eigenvalue analysis of two 12 MHz AT-cut resonators
A model of a 12 MHz AT-cut Quartz Resonator

Top electrode
Grounded
\[ f_R = 12.03559 \text{ MHz}, \quad Q\text{factor} = 1.49 \times 10^5 \]

Bottom electrode
• Grounded

Silicon rubber mounting, isotropic material:
\[ E = 2.0 \times 10^8 + j \times 2.0 \times 10^7 \text{ Pa} \]
\[ \nu = 0.33 \]
\[ \rho = 1050 \text{ kg/m}^3 \]
Resonators A, and B
12 MHz AT-cut Quartz Resonators

A: Blank: X=1627 um, Z=5715 um, Y=132.8 um
Electrode: X=1288 um, Z=3048 um, 0.5 um gold

B: Blank: X=1627 um, Z=2413 um, Y=132.3 um
Electrode: X=1422 um, Z=635 um, 0.5 um gold
BVD electrical parameters by eigenvalue analysis

1) Obtain from eigenvalue analysis
   a) $\omega_R$, real, short circuit resonance frequency, rad/s
   b) $\omega_I$, imaginary, short circuit resonance frequency, rad/s

2) $Q = \frac{\omega_R}{2\omega_I}$

3) $C_1 = \frac{I_m^2}{2\omega_R^2 E_{kin}}$ where $E_{kin} = \frac{\omega_R^2}{2} \int_V \rho |u|^2 dV$,
   and $I_m$ is the current over the top electrode

4) $R_1 = \frac{1}{\omega_R Q C_1}$ motional resistance

5) $L_1 = \frac{1}{C_1 \omega_R^2}$ motional inductance

6) $C_0 =$ the total charge over the 1V top electrode in an electrostatic problem, bottom electrode grounded
### 12 MHz AT-Cut Thickness Shear Resonator

<table>
<thead>
<tr>
<th>BVD parameters</th>
<th>Resonator A</th>
<th>Resonator B</th>
<th>Resonator A</th>
<th>Resonator B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvalue analysis</td>
<td>^1 Measured values</td>
<td>Eigenvalue analysis</td>
<td>^1 Measured values</td>
</tr>
<tr>
<td>$f_r$, MHz</td>
<td>12.03559</td>
<td>12.0</td>
<td>12.25664</td>
<td>12.0</td>
</tr>
<tr>
<td>$^2C_0$, pF</td>
<td>1.22</td>
<td>1.23</td>
<td>0.31</td>
<td>0.35</td>
</tr>
<tr>
<td>$C_1$, fF</td>
<td>5.41</td>
<td>5.7</td>
<td>1.38</td>
<td>1.8</td>
</tr>
<tr>
<td>$L_1$, mH</td>
<td>32.8</td>
<td>Not available</td>
<td>128.3</td>
<td>Not available</td>
</tr>
<tr>
<td>$R_1$, ohm</td>
<td>16.6</td>
<td>15</td>
<td>130</td>
<td>&lt; 1000</td>
</tr>
</tbody>
</table>

^1 Measured values provided by Statek, Inc.

^2 From piezo-electrostatic calculation
Frequency response analysis of quartz resonators with material dissipation and mounting substrate
Frequency response analysis for AT-Cut quartz with and without dissipation

Real Part of current plot
Frequency response analysis for AT-cut quartz plate with and without dissipation.

Imaginary part of current plot.
Study of a tuning fork, and determination of its equivalent electrical parameters from the finite element model admittance curve (frequency response analysis)
Finite element model admittance curve $Y = G + jB$

Conductance $G$, susceptance $B$, and admittance $Y$ curves

- **Conductance $G$, mho**
- **Admittance $Y$**
- **Susceptance $B$, mho**

Susceptance is a function of both $C_0$, and the parameters in the motional arm.

Conductance is independent of the static capacitance $C_0$. 

**Theory and Analysis of Quartz Crystal Resonators** slide # 25
Steps for extracting the BVD model parameters

1. The static capacitance $C_0$ must be obtained first. One electrode is grounded, while the other electrode is applied with 1 V. The magnitude of the charge over the 1 V electrode is $C_0$. 
Steps for extracting the BVD model parameters

2. Determine the resistance and reactance of the motional arm
   a) Calculate \( B_1 = B - \omega C_0 \)
   b) Calculate \( R_1 = \frac{G}{G^2 + B_1^2} \)
   c) Calculate \( X_1 = -\frac{B_1}{G^2 + B_1^2} \)
   d) Calculate \( \frac{dX_1}{df} \) by the central difference method:
      \[
      \left[ \frac{dX_1}{df} \right]_i = \frac{[X_1]_{i+1} - [X_1]_{i-1}}{f_{i+1} - f_{i-1}}
      \]
Steps for extracting the BVD model parameters

3. Note the series resonance $f_s$ at the maximum admittance $Y$

4. Obtain the value of $R_1$ at $f = f_s$

5. Obtain the value of $L_1$ at $f = f_s$

6. Obtain the value of $C_1$ at $f = f_s$

7. Obtain the value of $Q$ at $f = f_s$

Note:

$$L_1 = \frac{1}{4\pi} \frac{dX_1}{df} \bigg|_{f=f_s}$$

$$C_1 = \frac{1}{4\pi f_s^2} \frac{1}{L_1} \bigg|_{f=f_s}$$

$$Q = \frac{1}{2\pi f_s C_1 R_1} \bigg|_{f=f_s}$$
Results for a 2° Z-cut 163 KHz quartz tuning fork resonator

\[ C_0 = 0.3 \text{ pF} \]
Admittance curve of a tuning fork with base on a mounting support (no PML), 2° Z-cut quartz with Lamb & Richter damping. (Frequency range 140 KHz to 240KHz with a frequency increment of 200 Hz)

\[ C_0 = 0.29983 \text{ pF} \]

\[ Q = \frac{1}{2\pi f C_1 R_1} \]

Note that the curves for \( R_1, L_1, C_1, \) and \( Q \) are smooth functions, hence they are less sensitive to the frequency increments of \( Y \)
Admittance curve of a tuning fork with base on a mounting support (no PML), 2° Z-cut quartz with Lamb & Richter damping. (Frequency range 140 KHz to 240KHz with a frequency increment of 200 Hz)
Comparison of the BVD electrical parameters obtained from the FEM admittance curves with different frequency increments

<table>
<thead>
<tr>
<th>FEM admittance curve with frequency range 140 KHz to 240 KHz, and frequency increments of 200 Hz</th>
<th>FEM admittance curve with frequency range 163.6 KHz to 164.4 KHz, and frequency increments of 5 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_0 = 0.29983 \text{ pF} )</td>
<td>( C_0 = 0.29983 \text{ pF} )</td>
</tr>
<tr>
<td>( R_1 = 127.58 \text{ ohm} )</td>
<td>( R_1 = 126.12 \text{ ohm} )</td>
</tr>
<tr>
<td>( L_1 = 1357.3 \text{ H} )</td>
<td>( L_1 = 1350.6 \text{ H} )</td>
</tr>
<tr>
<td>( C_1 = 0.69957 \text{ fF} )</td>
<td>( C_1 = 0.69830 \text{ fF} )</td>
</tr>
<tr>
<td>( Q = 10.949\times10^6 )</td>
<td>( Q = 10.949\times10^6 )</td>
</tr>
</tbody>
</table>
Comparison of admittance curves of a tuning fork with base on a mounting support (no PML), $2^\circ$ Z-cut quartz with Lamb & Richter damping. (Frequency range 163.6 KHz to 164.4 KHz with a frequency increment of 5 Hz)

Comparison of the FEM admittance curve with the predicted BVD admittance curve

Predicted values of the BVD electrical parameters:
- $C_0 = 0.29983 \text{ pF}$
- $R_1 = 126.12 \text{ ohm}$
- $L_1 = 1350.6 \text{ H}$
- $C_1 = 0.69830 \text{ fF}$
- $Q = 10.949 \times 10^6$

Frequency, KHz

Theory and Analysis of Quartz Crystal Resonators  slide # 33
Effects of dissipation (PML) at the mounting supports on the BVD model parameters
Admittance curve of a tuning fork with base fixed on a mounting support (with PML), 2° Z-cut quartz with Lamb & Richter damping. (Frequency range 163.6 KHz to 164.4 KHz with a frequency increment of 5 Hz)

C1, Qfactor, L1, R1, and Y versus f

- \( f_s \) is the resonance frequency.
- \( C_0 = 0.29983 \) pF
- \( R_1 = 69481 \) ohm
- \( Q = 20017 \)
- \( L_1 = 1350.7 \) H
- \( C_1 = 0.69825 \) fF
Comparison of the effects of dissipation (PML) at the mounting support on the BVD electrical parameters

<table>
<thead>
<tr>
<th>No PML at the mounting support</th>
<th>With PML at the mounting support</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM admittance curve with</td>
<td>FEM admittance curve with</td>
</tr>
<tr>
<td>frequency range 163.6 KHz to</td>
<td>frequency range 163.6 KHz to</td>
</tr>
<tr>
<td>164.4 KHz, and frequency</td>
<td>164.4 KHz, and frequency</td>
</tr>
<tr>
<td>increments of 5 Hz</td>
<td>increments of 5 Hz</td>
</tr>
<tr>
<td>( C_0 = 0.29983 \text{ pF} )</td>
<td>( C_0 = 0.29983 \text{ pF} )</td>
</tr>
<tr>
<td>( R_1 = 126.12 \text{ ohm} )</td>
<td>( R_1 = 69481 \text{ ohm} )</td>
</tr>
<tr>
<td>( L_1 = 1350.6 \text{ H} )</td>
<td>( L_1 = 1350.7 \text{ H} )</td>
</tr>
<tr>
<td>( C_1 = 0.69830 \text{ fF} )</td>
<td>( C_1 = 0.69825 \text{ fF} )</td>
</tr>
<tr>
<td>( Q = 10.949 \times 10^6 )</td>
<td>( Q = 20017 )</td>
</tr>
</tbody>
</table>
Frequency response before mounting onto a base in the package

Frequency response after mounting onto a base in the package
The effects of packaging

Frequency response before packaging

Frequency spectrum of a plated blank

Frequency response after packaging

Frequency spectrum of a plated blank with support
FEM model with an energy sink
Two types of energy absorbing base could be modeled

- Base with acoustic loss as an energy sink

- Perfectly Matched Layer as an energy sink

No reflection at the interface
Perfect decay of the wave at bottom layer
13 MHz case

Energy trapping not good, large acoustic energy losses to the base

48 MHz case

Good energy trapping, minimal acoustic energy losses to the base
Frequency response analysis (13 MHz case)

Calculated admittance curve using model with viscosity loss only

Experimental result

Calculated admittance curve using model with viscosity loss and PML
Frequency response analysis (48 MHz case)

- Calculated admittance curve using the model with viscosity only
- Calculated admittance curve using the model with viscosity and PML
- Measured admittance curve

Admittance (mho)

Normalized Frequency

Theory and Analysis of Quartz Crystal Resonators slide # 43
Comparison of calculation results with experimental results

### 13MHz AT cut resonator

<table>
<thead>
<tr>
<th>Sample</th>
<th>$F$ (MHz)</th>
<th>$Q$ factor</th>
<th>$R_1$ (ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>13.253</td>
<td>1067</td>
<td>10730</td>
</tr>
<tr>
<td>With PML with viscosity</td>
<td>13.247</td>
<td>982</td>
<td>9256</td>
</tr>
<tr>
<td>Without PML with viscosity</td>
<td>13.247</td>
<td>630000</td>
<td>24</td>
</tr>
<tr>
<td>Fix condition at support</td>
<td>13.271</td>
<td>1000000</td>
<td>8.5</td>
</tr>
</tbody>
</table>

### 48MHz AT cut resonator

<table>
<thead>
<tr>
<th>Sample</th>
<th>$F$ (MHz)</th>
<th>$Q$ factor</th>
<th>$R_1$ (ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>47.997</td>
<td>69110</td>
<td>9.41</td>
</tr>
<tr>
<td>With PML with viscosity</td>
<td>48.165</td>
<td>199423</td>
<td>4.40</td>
</tr>
<tr>
<td>With PML, viscosity, lead resistance</td>
<td>48.165</td>
<td>175492</td>
<td>5.00</td>
</tr>
<tr>
<td>Without PML with viscosity</td>
<td>48.166</td>
<td>427159</td>
<td>2.03</td>
</tr>
</tbody>
</table>

Q factor can be estimated by consideration of both viscosity loss and supporting loss.
Q factors with viscosity loss

Q factor with viscosity + PML


Theory and Analysis of Quartz Crystal Resonators slide # 45
Analytical Considerations

There are no closed form analytical solutions for a 3-D resonator with finite dimensions.

Numerical methods are needed for designs.
Finite Element Method

Validation of the finite element method:

a) The AT-cut fundamental thickness shear mode

b) Frequency spectrum and comparison with Koga’s experimental data

c) Frequency-temperature characteristics and comparison with Sekimoto, et. al.’s experimental data
The AT-cut fundamental thickness shear mode

- Mode shape
- High current at the plate surfaces
Finite element model of a 1 MHz AT-cut quartz plate

- Cut angle 35.25 degrees about the digonal axis
- Dimensions: 16 mm X-length, 27 mm Z-length, 1.65 mm thickness
- Lagrange quadratic hexahedral elements used
- 3 x 35 x 50-element mesh
- 30 eigenfrequencies calculated centered about 1 MHz
Finite element mesh
Thickness shear mode shape
Comparison of the current at a plate surface of the thickness mode (mode 16) to that of other modes in the vicinity.
Frequency spectrum and comparison with Koga’s experimental data[1]

- Frequency spectrum = plot of resonant frequencies versus a parameter such as the length to thickness ratio (a/b)

Koga’s [1] frequency spectrum

Notes:
1. Koga measured the strong resonances in a quartz plate as a function of the X-length from 16 mm to 24 mm.
2. By meticulously “shaving” the X-length from 24 mm down to 16 mm, and carefully measuring and recording the strong resonances after each “shave”, he produced the frequency spectrum on the left.
3. He used a pair of air-gap electrodes to drive the plate.
4. It is more meaningful to plot resonance frequencies against the dimensionless length a/b ratio.
5. The red rectangle on the graph delineates the range of values to be compared with COMSOL model results.
Sorting the COMSOL model results

• The modal frequencies were calculated along with their relative ratio of shear (xy) strain energy to total strain energy and current.

• The modes were sorted into three groups:
  1. High ratio of shear (xy) strain energy to total strain energy and current modes
  2. High current modes (this represent the strong resonant modes measured by Koga [1])
  3. Other modes
Notes:
1. The COMSOL model produced more modes than measured by Koga.
2. All the COMSOL model modes are real but most are too faint to be detected by a pair of air-gap electrodes.
3. There are charge cancellations in most of the modes hence they could not be driven by a pair of electrodes on the plate major surfaces.
4. The red modal branch represents modes which are strong in current and ratio of shear (xy) strain energy to total energy, and it is usually the thickness shear modal branch.
5. The green modal branch represents the modes which are strong in current, and it is usually the strong resonances measured by Koga.
6. The blue modal branch represents the other modes which are usually not detected by Koga. These modes are usually not driven in a quartz plate resonator, however they are the spurious modes that could cause problems due to unsymmetrical electrodes & plate, mounting supports, nonlinear behavior, etc.
Frequency-temperature characteristics and comparison with Sekimoto, et. al.’s experimental data[2]

• Frequency deviations versus temperature

Finite element model of a 0.96 MHz AT-cut quartz plate

• Cut angle 35.25 degrees about the digonal axis
• Dimensions: 13.964 mm X-length, 7.000 mm Z-length, 1.737 mm thickness
• Lagrange quadratic tetrahedral elements used
• Mesh generated using free meshing of maximum element size 0.8 mm.
• 20 eigenfrequencies calculated about the 0.96 MHz
Thickness shear mode shape
Strong resonances (high current) as a function of temperature
Sorting of modes according to frequency, x-y shear energy and current

- In experiments, only the strong resonances could be measured readily. These are modes that yield relatively large currents.
- We sort the modes according to the charge on the top surface of the plate. Strong resonances have relatively large charges.
- We sort the modes according to the x-y shear energy. The thickness shear mode has both large charge and large x-y shear energy.
Comparison of f-T model with Sekimoto's data for Mode A

Frequency change, ppm, from 25°C

Temperature, C

Static f-T model

Sekimoto measured data
Comparison of f-T model with Sekimoto's data for the thickness shear mode

Frequency change, ppm, from 25°C

Temperature, C

-10 10 30 50 70

-120 -100 -80 -60 -40 -20 0 20 40 60 80

Static f-T model
Sekimoto measured data
Comparison of f-T model with Sekimoto's data for Mode B

- Frequency change, ppm, from 25°C
- Temperature, °C
Comparison of f-T model with Sekimoto's data for Mode C

Comparison of f-T model with Sekimoto's data for Mode C.
Thank you very much for participation!

Please send your comments, suggestions, and technical questions to

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