Analysis of Interdigital Surface Wave Transducers by Use of an Equivalent Circuit Model

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Invited Paper

Abstract—Immittance, transfer, and scattering characteristics are studied for acoustic surface wave transducers of the interdigital electrode form. Linear network models are used to represent the transducer as a chain of identical three-ports which are acoustically in cascade but electrically in parallel. Transducer operation at acoustic synchronism is described theoretically and compared to current experimental data for transducers operating at 100 MHz and fabricated on lithium niobate. Favorable lithium niobate configurations for efficient, broad-band transducer operation are given. Scattering characteristics as a function of electric load are discussed. Low values of acoustic reflection loss are predicted theoretically and observed experimentally when the electric load and transducer capacitance are in resonance. The frequency dependence of transducer radiation imittance is studied, and the response is found to be analogous to the response of an endfire antenna array.

I. INTRODUCTION

ACOUSTIC surface waves or Rayleigh, Love, and Stoneley waves [1]-[3], and their hybrids, such as piezoelectric carrier [4] waves and magnetoelastic waves [5], promise ultimate application in large scale microdistributed integrated circuits for analog and digital signal processing, requiring instantaneous bandwidths up to 1 GHz. Interest in surface waves stems from their short wavelengths, approximately 10\(^6\) times smaller than the electromagnetic wavelength at the same frequency, and the availability of crystalline substrates having linear dimensions of the order of a few centimeters, with attenuation per wavelength lower than for electromagnetic waves at frequencies below 2 GHz. In order to make full use of the potential applications available with surface waves, it is necessary to have efficient, broad-band transducers for conversion to and from electromagnetic energy. In this paper, we will consider the characteristics of input imittance, acoustic scattering, and electroacoustic conversion for the interdigital electrode transducer, which from recent experimental data [6] has been shown to be an efficient surface wave transducer.

Throughout the paper, interdigital transducers of the form shown in Fig. 1 will be described by linear network theory based on the analogous one-dimensional models given in Fig. 2. Section II describes the circuit characteristics of one periodic section of the transducer in terms of the equivalent circuit models and then outlines the combination of \( N \) sections to form a complete transducer. Section III is concerned with transducer operation at the synchronous frequency where the acoustic wavelength equals the interdigital period. Theoretical predictions of the circuit models are compared with diagnostic experimental data on synchronous radiation admittance and acoustic scattering properties. Quantitative predictions for the value of radiation \( Q \) are given and compared with experiment for a variety of cuts and propagation directions in lithium niobate, establishing favorable configurations. Section IV considers the frequency dependence of transducer characteristics. Predictions of the theory for radiation admittance are compared with measurements for \( Y \)-cut, \( Z \)-oriented lithium niobate and with an alternate approximate theory which regards the transducer as an endfire array.

II. EQUIVALENT CIRCUIT MODEL

We consider an interdigital transducer composed of \( N \) periodic sections of the form shown in Figs. 1 and 2. In principle, one could solve the boundary value equations for this configuration to find an admittance matrix relating the terminal quantities at the one electric and two acoustic ports. The resulting equations are difficult to solve because the problem is two-dimensional and contains a piezoelectric anisotropic substrate. As a first order approximation, we represent the interdigital periodic section by one of the analogous one-dimensional configurations shown in Fig. 2(b) and (c). Essentially, a pair of bulk wave transducers are arranged acoustically in cascade and electrically in parallel, such that the necessary electric field reversal is present. For the configuration of Fig. 2(b), which we shall call the “crossed-field” model, the applied electric field is normal to the acoustic propagation vector. The second configuration, shown in Fig. 2(c), termed the “in-line” model, is characterized by parallel electric field and propagation vectors. The analogy between either model and the true field distribution is appropriate in that the terminal electric and acoustic quantities have the same sign and physical symmetry. For a given piezoelectric with given cut and orientation, the choice between “crossed-field” and “in-line” models is made by eval-
The important advantage of the one-dimensional model is that each periodic section can be represented by the Mason equivalent circuit shown in Fig. 3. Following Berlincourt et al. [9], electric unit equivalents for the acoustic terminal and particle velocity are defined as

$$e_i = F_i / \phi$$
$$i_i = U_i / \phi$$

where $\phi = hC_v/2$ is regarded as the turns ratio of an acoustic-to-electric circuit transformer. These definitions allow the substrate characteristic mechanical impedance $Z_0$ to be expressed in electrical ohms by

$$R_0 = Z_0 / \phi^2 = 2\pi \frac{f_0}{\omega_0 C v k^2}$$

(3)

where $k$ is the electromechanical coupling constant, $C$ is the static electrode capacitance of one periodic section, and $\omega_0 = 2\pi f_0$ is the synchronism frequency defined by $f_0 = v/L$. Thus, three constants must be specified to describe the operation of one periodic section of the circuit model. The synchronism frequency is computed from the known surface velocity and periodic length and the capacitance found from well-known theories [10] or experimentally measured. More difficult to determine is the electromechanical coupling constant appropriate for a given surface wave configuration. As defined for bulk wave transducers [9], the coupling constant is the ratio of mutual elastic and dielectric energy to the geometric mean of elastic and dielectric self-energies. However, the value of $k$ can also be obtained by calculating the perturbation of wave velocity $\Delta v$ due to a change in the electric field boundary conditions. As an example, for the one-dimensional bulk wave configuration the fractional velocity change occurring when the piezoelectric field is shorted out is [9]

$$\frac{\Delta v}{v} = 1 - \sqrt{1 + k^2} \approx -\frac{1}{2} k^2.$$

(4)

For the surface wave configuration we consider the velocity perturbation when the tangential electric field is shorted out at the piezoelectric surface. Physically, this perturbation can be obtained by coating the piezoelectric surface with a very thin metal film. Approximate calculations of $\Delta v/v$ for CdS, ZnO, PZT, LiTaO$_3$, and LiNbO$_3$ substrates have been re-
ported by Tseng [11] and Ingebrigtsen [12]. Exact calculations for LiNbO₃ have been presented by Campbell and Jones [13]. Since the field distributions for the interdigital and metal-coated configurations are not identical, the relation between transducer coupling constant and velocity perturbation is more complex than the simple expression given in (4) for the bulk wave example. However, an analogous relation can be defined by introducing a filling factor $F$ so that

$$k^2 = 2F \left| \frac{\Delta \psi}{v} \right|.$$  

Measurements of transducer impedance, described in Section III, show that $F = 1.0 \pm 0.2$ for transducers deposited on lithium niobate with equal electrode spacing and width.

Having formulated the equivalent circuit for one periodic section, the admittance matrix for the entire transducer can now be calculated using the block diagram shown in Fig. 4. The transducer 3-port equation is written as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} [Y] \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$  

where the admittance matrix $[Y]$ is shown in Appendix II to have the form

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12} & Y_{11} & -Y_{13} \\ Y_{13} & -Y_{13} & Y_{33} \end{bmatrix}. \tag{7}$$

The matrix $[Y]$ has four independent elements and exhibits the expected symmetry of the two acoustic ports with respect to the electric port. Thus, $Y_{11} = Y_{22}$, $Y_{12} = Y_{21}$, and $Y_{13} = -Y_{32}$. Since a voltage applied at port 3 will result in stress of the same sign at ports 1 and 2, the change of sign between $Y_{13}$ and $Y_{32}$ is necessary to ensure that acoustic power flows symmetrically away from the transducer. As shown in Appendix II, the matrix elements for the "crossed-field" model are

$$Y_{11} = -jG_0 \cot N\theta$$
$$Y_{12} = jG_0 \csc N\theta$$
$$Y_{13} = -jG_0 \tan \frac{\theta}{4}$$
$$Y_{33} = j\omega C_T + 4jNG_0 \tan \frac{\theta}{4} \tag{8}$$

where $G_0 = R_0^{-1}$, $N$ is the transducer length measured in interdigital periods, and $C_T = NC$ is the total electrode capacitance. For the "in-line" model

$$Y_{11} = -S_{11}/S_{12}$$
$$Y_{12} = 1/S_{12}$$
$$Y_{13} = -jG_0 \tan \frac{\theta}{4}$$
$$Y_{33} = \frac{j\omega C_T}{1 - 2X \tan \frac{\theta}{4}} \tag{9}$$

where $X = 2G_0/\omega C_T$, and the matrix $[S]$, defined in Appendix II, is a complex function of $N$, $\theta$, $X$, and $G_0$.

Within the restrictions outlined above, the 3-port equation (6) provides a complete description of interdigital transducer operation. Impedance, transfer, and scattering characteristics may be calculated by use of standard network theory [14]. In general, the matrix elements are sufficiently complex that calculations must be done with the aid of a digital computer. However, it is shown in Section III for frequencies near acoustic synchronism that the admittance matrix for either model becomes simple enough to allow hand calculations.

### III. Transducer Performance at Acoustic Synchronism

#### A. Transducer Admittance Matrix

A consideration of the symmetry and periodicity of the interdigital transducer leads to the conclusion that maximum acoustic response, for a given applied voltage, must occur for frequencies at or near acoustic synchronism. In this section we compare the synchronous performance of the "in-line" and "crossed-field" models with measured data for transducers fabricated on lithium niobate. The prediction made in Appendix I that the "crossed-field" model is more suitable for transducers on $Y$-cut, $Z$-propagating lithium niobate is verified.

As shown in Appendix II for $\omega = \omega_0$, the admittance matrix for the "in-line" model has the simple form

$$[Y] = \frac{j\omega C_T}{16} \begin{bmatrix} 1 & 1 & 4 \\ N & N & -4 \\ 4 & -4 & 0 \end{bmatrix} \tag{10}$$

For the "crossed-field" model the matrix elements become infinite at synchronism, since $\theta = 2\pi(\omega_0/\omega) = 2\pi$ in (8). However, the impedance and transfer functions obtained from $[Y]$ remain finite and may be calculated by expanding the matrix for frequencies very near synchronism. By setting $\theta = 2\pi + \delta$ and expanding to the first order in $\delta$, the "crossed-
field” matrix becomes

$$[Y] \approx \frac{\mathcal{G}_0}{\delta} \begin{bmatrix} -1 & 1 & \cdots & 1 & -1 \\ \frac{1}{N} & \frac{1}{N} & \cdots & \frac{1}{N} & -\frac{1}{N} \\ -4 & -4 & \cdots & -4 & -4 \end{bmatrix} \frac{1}{4 - 4 - 16N + \frac{\omega C_T}{G_0}}$$

(11)

The matrix descriptions of the two models are seen to be similar, but with important differences in element \(Y_{ss}\) and overall magnitude.

B. Transducer Electrical Immittance

Input immittance of the interdigital transducer may be characterized by a circuit consisting of the total electrode capacitance in parallel or series with a radiation immittance representing acoustic wave excitation. The resultant electrical behavior can be represented by either the series or shunt circuits shown in Fig. 5. We shall see that the performance of the “crossed-field” model is well-suited for the shunt circuit whereas the “in-line” model is represented best by the series circuit.

We assume that the transducer radiates into a medium of infinite extent. Experimentally, an infinite medium is approximated either by using a short RF pulse measurement or, if CW signals are used, by loading the acoustic surface with a low-reflection absorber, such as black wax, to provide acoustic termination. The condition of infinite acoustic medium is created for the transducer model by connecting the characteristic impedance \(R_0\) across ports 1 and 2 in Fig. 4. For the present discussion the effect of electrical dissipation loss in the interdigital electrodes will be neglected.

For the “in-line” model the electrical immittance may be obtained from (10) as the series impedance

$$Z_s(\omega) = \frac{E_s}{I_s} = \mathcal{R}_a + \frac{1}{j\omega C_T}$$

(12)

where \(\mathcal{R}_a = R_a(\omega) = \frac{4}{\pi} k^2(1/\omega C_T)\). Thus, the series circuit of Fig. 5(a) is convenient for the “in-line” model since the reactance of the transducer capacitance \(C_T\) appears as a series term. We note that the series radiation reactance \(X_s(\omega)\) is zero and that the radiation resistance \(\mathcal{R}_a\) is independent of \(N\), the transducer length measured in interdigital periods.

As shown in (11), the immittance for the “crossed-field” model is conveniently expressed as the admittance

$$Y_s(\omega) = \frac{I_s}{E_s} = \mathcal{G}_a + j\omega C_T,$$

(13)

where \(\mathcal{G}_a = G_a(\omega) = \frac{4}{\pi} k^2(\omega C_T)N^2\). The form of (13) shows that the shunt circuit of Fig. 5(b) is most suitable for the “crossed-field” model and that \(B_s(\omega)\) is zero. We further note that the radiation conductance, \(\mathcal{G}_a\), is proportional to \(N^2\).

Although the synchronous immittance behavior of the two models appears to be quite different as described above, we note that

if the condition \([(4/\pi)k^2N] < 1\) is valid. For transducers deposited on the more active piezoelectrics, the factor \([(4/\pi)k^2N]^4\) can be made significant compared to unity, so that a distinction between the two models can be made experimentally. A straightforward procedure is to measure the synchronous admittance or impedance as a function of \(N\) and compare with (12) and (13). For example, the data shown in Fig. 6 for transducers fabricated on Y-cut, Z-propagating lithium niobate, demonstrates “crossed-field” behavior. The transducer capacitance, measured at low frequency to approximate static conditions, was first studied to show that \(C_T\) is proportional to \(N\) [see Fig. 6(b)] as has been implicitly assumed in the foregoing analysis. The synchro-nism admittance was then measured and \(j\omega C_T\) subtracted, yielding the radiation admittance \(G_a(\omega) + jB_a(\omega)\). For transducers with \(N = 5, 10, 15, 20, 25,\) and 30 the value of
$B_a(\omega_0)$ was found to be zero within the five percent experimental error. Further, the radiation conductance, shown in Fig. 6(a), was found to be proportional to $N^2$ as predicted by the "crossed-field" model.

C. Favorable Cuts and Propagation Directions for Lithium Niobate

For efficient transducer operation over maximum bandwidth, network matching requirements call for minimizing the synchronous reactance or susceptance while matching the radiation resistance or conductance to the generator. For simple resonant tuning circuits, this is equivalent to minimizing the radiation $Q$ defined by

$$Q_r = \frac{1}{\omega_0 C_T R_a} \text{ "in-line" model}$$

$$Q_r = \frac{\omega C_T / G_a}{\omega / V} \text{ "crossed-field" model.}$$

(15)

It follows for either model that

$$\langle NQ_r \rangle^{-1} = \frac{4}{\pi} \frac{k^2}{\pi} F \frac{\Delta v}{v} I^2,$$  

(16)

demonstrating that $NQ_r$ depends only on the effective coupling constant. In (16), a linear dependence of $Q_r^{-1}$ on $N$ is predicted. This has been shown experimentally for $YZ$ (Y-cut, Z-propagating) lithium niobate as illustrated by the data shown in Fig. 6(c).

In Fig. 7, measurements of $\langle NQ_r \rangle^{-1}$ are plotted versus $\Delta v/v$ for transducers fabricated for six distinct cuts and propagation directions in lithium niobate which have collinear phase and group velocities. The values of $\Delta v/v$ are those calculated by Campbell and Jones [13]. In each case, $N=20$ and the periodic length $(L)$ is 32.5 microns, but the condition of synchronism requires a different $\omega_0$ for each substrate configuration. The linear graph of (16) which is plotted in Fig. 7 for $F=1$ shows that the filling factor is near unity for most configurations. With exception of the $YZ$ configuration, $F=1.0 \pm 0.2$. It is concluded that $YZ$ and $XZ$ lithium niobate configurations are most favorable for efficient transducer operation.

D. Scattering and Transmission Characteristics as a Function of Electrical Load

A detailed knowledge of the scattering and transmission characteristics of interdigital transducers is necessary in the design of delay line taps for use in signal processing. We consider the transducer as a tap whose scattering characteristics vary as a function of the electrical termination. It will be convenient to define a set of power scattering coefficients as follows.

$$P_{ij} = \frac{P_i}{(P_{avail})_j},$$

(17)

where $P_i$ is the power transmitted or reflected from port $i$ and $(P_{avail})_j$ is the power available from a matched generator at port $j$. Thus, $p_{ij}$ is the fraction of power reflected when power is incident at port $i$, and $p_{ij}(i\neq j)$ is the fraction of power transmitted from port $i$ when power is incident at port $j$. From reciprocity and the symmetry of the interdigital transducer, as shown in Fig. 4, it follows that $p_{11}=p_{22}$, $p_{12}=p_{21}$ and $p_{ij}=p_{ji}$. Experimentally, power ratios are usually measured in dB; thus, we define scattering loss in dB by

$$L_{ij} = -10 \log (p_{ij})$$

(18)

where $L_{ii}$ is the reflection loss at port $i$ and $L_{ij}(i\neq j)$ is the transmission loss from port $j$ to port $i$.

In Fig. 8, an acoustic wave is incident at port 1, port 2 is acoustically terminated, and port 3 is electrically terminated in $Z_L=1/Y_L$. For the moment, let us assume that $Z_L=jX_L$ (or $Y_L=jB_L$), a purely reactive load. From (6), (10), and (11) at synchronism, the scattering coefficients are found to be

$$p_{11} = \frac{1}{1 + a^2}$$
$$p_{21} = \frac{a^2}{1 + a^2}$$
$$p_{31} = 0$$

(19)

where the quantity $a$ is defined separately for the two models:

$$a = \left(\frac{X_L - \frac{1}{\omega_0 C_T}}{(B_L + \omega_0 C_T)/G_a}\right) \text{ "in-line"}$$
$$a = \left(\frac{X_L - \frac{1}{\omega_0 C_T}}{(B_L + \omega_0 C_T)/G_a}\right) \text{ "crossed-field".}$$

(20)

Thus, $a$ represents the normalized sum of susceptance or reactance of the applied load and transducer capacitance.

The most striking feature of the above analysis is that complete acoustic reflection ($L_{11}=0$, $L_{21}=\infty$) is predicted when the electrical load resonates the transducer capacitance ($a=0$). Data illustrating measured scattering loss is shown in Fig. 9. A delay-line configuration of three identical $N=15$ transducers spaced by 1 cm on $YZ$ lithium niobate was used. Scattering loss for the center transducer was found with the
end transducers performing as identical transmitting and receiving elements, and zero transmission loss for the center transducer was assumed to occur for minimum delay line insertion loss. Within the ±0.25 dB measurement uncertainty, the reflection loss is zero at \( a = (B_L + \omega_C T)/\tilde{G}_a = 0 \), as shown in Fig. 9. The data are in good agreement over the range \(|a| < 1\) with the theoretical curve calculated from the "crossed-field" model. The apparent disagreement between theory and experiment for \(|a| > 1\) is the result of experimental measurement at a frequency slightly different from synchronism, as is explained in detail in [18].

In the design of delay line taps, \( Z_L \) absorbs a controlled amount of power while electrically resonating the transducer capacity. For the "in-line" model, the convenient form is a series load, \( Z_L = R_L + jX_L \), with \( X_L = 1/\omega_C T \). The corresponding situation for the "crossed-field" model is a shunt load \( Y_L = G_L + jB_L \) with \( B_L = -\omega_C T \). The power scattering coefficients for either model at synchronism are

\[
\begin{align*}
\rho_{11} &= \frac{1}{(1 + b)^2} \\
\rho_{21} &= \frac{b^2}{(1 + b)^2} \\
\rho_{31} &= \frac{2b}{(1 + b)^2}
\end{align*}
\]

(21)

where

\[
b = \begin{cases} 
R_L/\tilde{R}_a & \text{"in-line" model} \\
G_L/\tilde{G}_a & \text{"crossed-field" model.}
\end{cases}
\]

(22)

Equations (21) and (22) show that minimum acoustic-to-electric conversion loss obtains for a conjugate matched load \((a = 0, b = 1)\), with minimum conversion loss equal to 3 dB and corresponding acoustic reflection and transmission losses equal to 6 dB. The same conclusions can also be reached by scattering matrix analysis for a lossless, symmetrical, 3-port network [15].

The configuration of three transducers on \( YZ \) lithium niobate described earlier was used again to measure the scattering losses for shunt resonant electrical loads, and Fig. 10 compares the data obtained with theoretical curves calculated from the "crossed-field" model. Good agreement is seen for \( b = G_L/\tilde{G}_a \) in the range of 0.5-2.0, and the conjugate matched condition \((L_{31} = 3 \text{ dB}, \ L_{11} = L_{31} = 6 \text{ dB})\) is verified within the ±0.25 dB measurement error. As in the reactive scattering measurements, the apparent disagreement for extreme values of load is explained in [18].

The experiment was also performed for series resonant loads and compared to theory for the "in-line" model. Except for the conjugate matched load, which again gave \( L_{31} = 3 \text{ dB}, \ L_{11} = L_{31} = 6 \text{ dB} \), theory and experiment were in poor agreement, lending further support to the "crossed-field" model as the better representation for the \( YZ \) lithium niobate configuration.

### IV. Frequency Dependence of Acoustic Radiation

In terms of the one-dimensional models, the frequency dependence of transducer acoustic radiation is described by the frequency response of the radiation immittances defined in Fig. 5. Again we assume that the transducer radiates into a medium of infinite extent. For the "in-line" model a straightforward application of (6), (7), and (9) reveals that the radiation immittance is a series impedance \( R_0(\omega) + jX_0(\omega) \), as shown in Fig. 5(a), but the calculation is sufficiently complex that numerical evaluation is necessary. For the "crossed-field" model the relatively simpler matrix elements given in (8) allow an analytic description. A shunt admittance \( G_0(\omega) + jB_0(\omega) \) is found in this case with

\[
G_0(\omega) = 2G_0 \left[ \tan \frac{\theta}{4} \sin \frac{N\theta}{2} \right]^2
\]

(23)

and

\[
B_0(\omega) = G_0 \tan \frac{\theta}{4} \left[ 4N + \tan \frac{\theta}{4} \sin N\theta \right].
\]

(24)

For frequencies near acoustic synchronism, (23) and (24) are approximately given by

\[
G_0(\omega) \cong \tilde{G}_0 \left( \frac{\sin x}{x} \right)^3
\]

(25)
Synchronism conductance defined in Section III. The approx­
etic response of an endfire antenna array. The (sin
imate expressions are accurate within ten percent for
factor in (25) confirms this analogy for the "crossed-field"
configuration.

Fig. 11. Measured radiation admittance for an N=15 transducer on
YZ lithium niobate compared with theoretical curves calculated
from the "crossed-field" model.

and

\[ B_x(\omega) \cong \frac{\sin 2x - 2x}{2x^2} \quad (26) \]

where \( x = \frac{N\pi(\omega - \omega_0)}{\omega_0} \) and \( \hat{G}_s = \frac{(4/\pi)k^2(\omega_0 C_s)N^2}{2} \) is the
synchronism conductance defined in Section III. The approx­
imate expressions are accurate within ten percent for
(\( \omega - \omega_0 \)). As conjectured by White [16], the
periodic configuration of the interdigital transducer should
lead to an acoustic response analogous to the electromagnetic
response of an endfire antenna array. The \( \sin x/x \) factor in (25) confirms this analogy for the "crossed-field"
model. A similar analysis for the "in-line" model shows that
the quantities \( R_x(\omega) \) and \( X_x(\omega) \) have the same response functions as the "crossed-field" \( G_x(\omega) \) and \( B_x(\omega) \), but with an
additional multiplier of \( (\omega_0/\omega)^2 \).

The prediction made in Section III that maximum \( G_x(\omega) \)
and zero \( B_x(\omega) \) occur at synchronism for the "crossed-field"
model is verified by (25) and (26). Further, the fractional band-
width between zeros for the central lobe of radiation conduc­tance is seen to be \( 2/N \). In Fig. 11 the experimentally measured radiation admittance for an \( N=15 \) transducer fabricated on YZ lithium niobate is compared with theoretical
curves calculated from (25) and (26). The capacitance for
one periodic section is calculated from the measured
transducer capacitance \( C_s = C_T/N \). The synchronous
frequency is found from the peak of the \( G_x(\omega) \) data, and the
effective coupling constant is estimated from Fig. 7 as \( (4/\pi)k^2
= 2.3 \Delta v/\nu \) = 0.056. The agreement of theory and exper­
iment is within ten percent, again supporting the "crossed-field"
model as a good representation for the YZ transducer
configuration.

V. CONCLUSIONS

The essential 3-port characteristics of the interdigital
transducer have been described by linear equivalent circuits
based on approximate one-dimensional models. Two criteria
have been presented to determine which of the "crossed-field"
or "in-line" models is more appropriate for a specific
piezoelectric configuration. The theoretical discussion in
Appendix I gives a technique for separating the Rayleigh
wave mutual stored energy into terms associated with compo­
nents of the electric field that are normal and parallel to the
surface. Predominance of the normal field term indicates
that the "crossed-field" model is the more suitable of the two
models. Conversely, the "in-line" model is selected when the
parallel field term is largest. Experimentally, measurements
of the synchronous electric imittance quantities \( \hat{R}_x \) or \( \hat{G}_x \)
can be used to distinguish between the two models, when
\( (4/\pi)k^2N \) is of order unity. Further support for the chosen
model is obtained from 3-port scattering measurements and
from the frequency dependence of transducer electrical input
imittance. The evaluation of these criteria for transducers
fabricated on the YZ configuration of lithium niobate has
shown that the transducer 3-port properties are well-repre­
dented by the "crossed-field" model.

APPENDIX I

EVALUATION OF COUPLING ENERGY

Following Berlincourt et al. [9], the time average energy
density of a linear piezoelectric medium, neglecting magnetic
terms, is expressed in vector notation [7] by

\[ w = \frac{1}{2} \{ T^\ast \cdot S + E^\ast \cdot D \}. \]

Upon substitution of the appropriate constitutive relations,
(27) becomes

\[ w = \frac{1}{2} T^\ast \cdot S + \frac{1}{2} \{ E^\ast \cdot d \cdot T \} + \frac{1}{2} \{ E^\ast \cdot d \cdot E \} = w_e + 2w_m + w_d \]

where \( w_e, w_m, \) and \( w_d \) are the elastic, mutual, and dielectric
energy density terms, respectively. The mutual energy density
\( w_m \) is of direct importance to the piezoelectric excitation prob­
lem because it appears as a distributed source term in the
normal mode theory of piezoelectric waveguides [7]. With
the aid of the material constant identities,

\[ c^R \cdot d = c \quad e^R \cdot d = e^T - e^P, \]

the \( w_m \) can be conveniently expressed in terms of \( S \) and \( E \)
as

\[ w_m = \frac{1}{2} \text{Re} \{ S \cdot e \cdot E \} + \frac{1}{2} \{ E^\ast \cdot \{ e^P - e^T \} \cdot E \}. \]

For a Rayleigh wave propagating on the surface of a piezo­
electric half space, (30) can then be written as the sum

\[ w_m = w_{\perp} + w_{\parallel}, \]

where \( w_{\perp} \) results from the component of \( E \) perpendicular to
the direction of propagation and \( w_{\parallel} \) arises from the parallel
component of \( E \). To assess the relative Rayleigh wave cou­
pling energy associated with the two electric field components,
we define the ratio as

\[ r = \frac{W_{\perp}}{W_{\parallel}} = \frac{\int_\nu w_{\perp}dV}{\int_\nu w_{\parallel}dV}. \]

In principle, one could perform the above analysis for the
electroded surface configuration of an interdigital transducer,
but the boundary conditions make the analysis difficult.
Here we assume that the value of \( r \) will not change appreci­
ably from free to electroded surface conditions, since the
integrals of field quantities remain constant within a few per­
cent even for relatively strong piezoelectrics. The calcula­
tions of Campbell and Jones [13] illustrate this behavior for lithium niobate.

As a specific example, we consider the case of Rayleigh wave propagation in the Z-direction on Y-cut lithium niobate. Equation (32) becomes

\[
y_{11} = -jG_0 \tan \frac{\theta}{4}
\]

\[
y_{12} = jG_0 \tan \frac{\theta}{4} + \omega C_s \frac{\theta}{4}
\]

(36)

With the material constants given by Warner et al. [17] and field quantities \(S\) and \(E\) derived from potential and particle displacement plots supplied by Jones [13], (33) gives \(r = 10\).

APPENDIX II

TRANSDUCER ADMITTANCE MATRIX

Here we obtain the 3-port admittance matrix of the transducer by first finding the matrix of one interdigital period and then applying a cascading formalism. The admittance matrix for one section is easily found by standard circuit analysis, with reference to Fig. 3. The appropriate form, showing the symmetry, is

\[
[y] = \begin{bmatrix}
y_{11} & y_{12} & y_{13} \\
y_{12} & y_{11} - y_{13} \\
y_{13} & -y_{12} & y_{33}
\end{bmatrix}.
\]

(34)

The values of the four independent elements differ accordingly as the negative capacitor in Fig. 3 is either short-circuited or included, representing the “in-line” or “crossed-field” distributions, respectively. They are as follows:

1) “in-line” model

\[
y_{11} = -jG_0 \cot \frac{\theta}{4} \left( x - \cot \frac{\theta}{2} \right) \left[ 2 - \frac{\left( x - \csc \frac{\theta}{2} \right)^2}{(x - \cot \frac{\theta}{2})^2} \right]
\]

\[
y_{12} = jG_0 \cot \frac{\theta}{4} \left( x - \csc \frac{\theta}{2} \right) \frac{2}{2x - \cot \frac{\theta}{4}} \left( x - \cot \frac{\theta}{2} \right)
\]

\[
y_{13} = -jG_0 \tan \frac{\theta}{4} \frac{1 - 2x \tan \frac{\theta}{4}}{1 - 2x \tan \theta}
\]

\[
y_{33} = \frac{j\omega C_s}{1 - 2x \tan \frac{\theta}{4}}
\]

(35)

where \(G_0 = R_0^{-1}, x = 2G_0/\omega C_s, \) and \(\theta = 2\pi/\omega_0.\)

2) “crossed-field” model

\[
y_{11} = -jG_0 \cot \theta
\]

\[
y_{12} = jG_0 \csc \theta
\]

The 3-port matrix for the entire transducer is found by connecting the \(N\) periodic sections in cascade acoustically and in parallel electrically, as is shown in Fig. 4. Since the sections are identical we have the recursion relation

\[
\begin{bmatrix}
i_{n-1} \\
-i_n \\
i_{2n}
\end{bmatrix} = \begin{bmatrix}
e_{n-1} \\
e_n \\
e_{2n}
\end{bmatrix}
\]

(37)

The total transducer current is the sum of currents flowing into the \(N\) sections. With the help of network symmetry and reciprocity the total current is

\[
I_a = y_{11}e_0 + y_{12}e_N + y_{23} \sum_{n=1}^{N} e_{2n}.
\]

Application of the boundary conditions \(e_0 = E_1, e_N = E_3, e_{2n} = E_2\) and network symmetry show that

\[
Y_{12} = Y_{23} = 0
\]

\[
Y_{31} = Y_{13} = \frac{1}{y_{12}}
\]

(39)

\[
Y_{33} = N y_{33}.
\]

From (37) we find a second recursion relation

\[
\begin{bmatrix}
e_n \\
i_n
\end{bmatrix} = \begin{bmatrix}e_{n-1} \\
i_{n-1}
\end{bmatrix} + \begin{bmatrix}d_1 \\
d_2
\end{bmatrix} E_3
\]

(40)

where

\[
[R] = \begin{bmatrix}
\frac{y_{11}}{1 - y_{11}} & -1 & 1 \\
-y_{11} & y_{11} \cdot y_{12}^2 - y_{12}^2 & -y_{11}
\end{bmatrix}
\]

(41)

and \(d_1\) and \(d_2\) are also functions of \(y_{23}\) but will not be needed here. Applying (41) \(N\) times gives

\[
\begin{bmatrix}
e_N \\
i_N
\end{bmatrix} = [R]^{N-1} \begin{bmatrix}
e_0 \\
i_0
\end{bmatrix} + \sum_{n=0}^{N-1} [R]^n \begin{bmatrix}d_1 \\
d_2
\end{bmatrix} E_3.
\]

(42)

Solving (42) for \(i_N\) and \(i_0\) and again applying the boundary conditions gives

\[
Y_{11} = Y_{23} = -S_{11}/S_{12}
\]

\[
Y_{12} = Y_{21} = 1/S_{12}
\]

(43)

where \([S] = [R]^N\). Equations (39) and (43) summarize the admittance parameters for the entire transducer. For the “crossed-field” model the recursion matrix becomes

\[
[R] = \begin{bmatrix}
\cos \theta & -jG_0 \sin \theta \\
-jG_0 \sin \theta & \cos \theta
\end{bmatrix}
\]

(44)
which is the familiar circuit matrix for a transmission line of impedance $R_0$. The matrix $[S]$ is then obtained by simply replacing $\theta$ in $[R]$ by $N\theta$.

For frequencies near acoustic synchronism ($\theta = 2\pi \omega/\omega_0 \leq 2\pi$), the admittance matrix for both “in-line” and “crossed-field” models can be reduced to much simpler form. In particular, the matrix of one periodic section for the “in-line” model at $\theta = 2\pi$ becomes

$$[y] = \frac{j\omega C_s}{16} \begin{bmatrix} -1 & 1 & 4 \\ 1 & -1 & -4 \\ 4 & -4 & 0 \end{bmatrix}.$$  (45)

It can be seen from (45) that $[R]$ is in canonical form so that the transducer matrix is

$$[Y] = \frac{j\omega C_s}{16} \begin{bmatrix} -1 & 1 & 4 \\ 1 & -1 & -4 \\ 4 & -4 & 0 \end{bmatrix}.  \ (46)$$

For the “crossed-field” model the matrix elements are infinite at $\theta = 2\pi$, but as shown in Section III, the matrix can be simplified by writing $\theta = 2\pi + \delta$ and expanding the elements to the first order in $\delta$. The result is given in (11).

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