Abstract — The first- and second-order statistics of envelope detected ultrasound (US) B-mode images for the case of a scattering phantom with many scatterers per resolution cell have been previously derived. These characteristics are integrated over the region of a simulated focal (disk) lesion and the signal-to-noise ratio (SNR) for lesion detectability is obtained. This SNR requires the average number of independent speckle cells over the lesion area (analogous to the number of x-ray photons over the lesion area in incoherent light or x-ray imaging). This number is obtained from our autocorrelation analysis (second-order statistics). By setting the SNR expression equal to the threshold value \( SNR_T \) required to detect a lesion in the presence of speckle noise, the dependence of lesion contrast on lesion diameter at threshold is found, i.e., the contrast/detail function. This is a simple inverse relation for ideal observers of US B-scans. It also found that the contrast/detail results for envelope detection in diagnostic ultrasound are almost identical with the results for square law detection (the usual laser case) with the latter serving as an upper limit for performance in lesion detection. Finally, the results of human observer performance using a contrast/detail phantom are compared with the predictions for optimal or ideal performance. The results are comparable with results for photon imaging systems, with values of the SNR at threshold in the neighborhood of 2-3.

I. INTRODUCTION

An essential task in all medical imaging is the detection of focal lesions against background tissue within individual organs. In diagnostic ultrasound, examples of this problem are the detection of breast masses, focal lesions in the liver, or infarcted regions of myocardium. The most realistic technique for the measurement of an imaging system's capability to detect such lesions is an analysis of receiver operating characteristic (ROC) curves derived using clinical images [1]. However, it often requires months to collect a sufficient number of images with known “ground truth,” and perhaps weeks to carry out the viewing and scoring. A laboratory method of evaluation, the so-called “contrast/detail analysis” of images of phantoms, has been found to be an efficient means of analysis when only an estimate of system physical performance is required, and has been used extensively in measurements of radiographic [2] and computed tomographic (CT) imaging systems [3], [4], as well as nonmedical optical imaging systems [5], [6]. The technique yields the dependence of lesion contrast on lesion diameter at the observer's threshold for lesion detection. This experimentally determined contrast/detail function often departs from what is expected theoretically for ideal observers, indicating the possibility of improving performance by image processing.

Contrast/detail analysis has recently been extended to diagnostic ultrasound by Smith and Lopez [7] with the development of a new phantom. Fig. 1 shows a schematic of the phantom, a block of tissue mimicking gel, containing a row of conical inserts, each consisting of a different tissue simulating material. Cross-sectional ultrasound scans, perpendicular to the axes of the cones, at various positions along the lengths of the cones, result in images of disks of constant diameter but varying contrast in steps covering a range of \(-20 \text{ dB}\) relative to the background. The diameter of the disks varies from 1 to 18 mm and is determined by the scanning plane position relative to the base of the cones. Fig. 2 is a composite photograph of eight image planes of a contrast/detail phantom using an ADR 2130 scanner at 3.5 MHz. Note the changing contrast of the disks across the image and the decreasing diameters of the targets down the image. Observer evaluation of such images enables the measurement of threshold detectability of lesions as a function of object contrast and diameter. The results of observer evaluations of the films included in Fig. 2 are shown in Fig. 3(a) as a plot of the object contrast, relative
Fig. 2. Composite photograph of eight image planes of contrast/detail phantom imaged with ADR Model 2130 scanner at 3.5 MHz.

to the background, versus the diameter of the disks at the threshold of detection by the observers. Analogous results for a Unirad GZD scanner at 3.5 MHz are shown in Fig. 3(b). The figures show that at high contrasts (>4 dB), the detection capability of the scanner is only weakly dependent on object contrast and approaches the two point spatial resolution of the imaging system in the simulated tissue, included in the plots at a contrast of 20 dB (more in Section IV). For low contrast objects (echo amplitude <4 dB relative to background) the diameter for threshold detection is strongly dependent on object contrast.

These results are similar to analogous data previously reported for radiographic systems and CT by Cohen et al. [2]–[4]. Several investigations have been made to predict theoretically the ideal contrast/detail relationships for these imaging modalities [8]–[10] based on an understanding of the spatial resolution and the statistics of the noise for a given imaging technique. This paper describes the extension of these ideas to ultrasound B-scans.

In diagnostic ultrasound (US) the spatial resolution or point response is well-understood and depends on diffraction and the frequency characteristics of the transducer/scanner. However, an understanding of the image noise of US B-scans is still incomplete. The texture in the image of parenchymal tissue may be viewed as image signal or as undesirable noise, a speckle interference pattern. In this paper the texture will be treated as coherent speckle, and an analysis of diagnostic ultrasound contrast/detail detection in the presence of speckle will be developed based on stochastic signal and noise analysis. The discussion will be limited to lesion detection about and beyond
the depth of the transducer focal region in a phantom consisting of many point scatterers. Here we shall use the results of the previous paper in this issue [11]. First, the envelope detected B-mode signal \( V \) is characterized by the Rayleigh probability density function,

\[
p(V) = \frac{V}{\psi} \exp \left(-\frac{V^2}{2\psi}\right), \quad V \geq 0,
\]

\[
= 0, \quad \text{otherwise},
\]

and the parameter \( \psi \) depends on the phantom’s mean-square scattering strength. Second, the speckle in the image of such a phantom carries only information about the transducer and its focusing pattern. The speckle cell size is comparable to the resolution cell size; therefore, in the axial direction it is an inverse function of the pulse bandwidth, and in the lateral direction it is proportional to the beamwidth, increasing with increasing range. Results from the previous paper that will be required here are summarized in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Phasor Magnitude</th>
<th>Phasor Intensity</th>
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<tbody>
<tr>
<td>Lateral Direction*</td>
<td>( S_{cz} = 0.87 \lambda z_0/D )</td>
</tr>
<tr>
<td>Range Direction†</td>
<td>( S_{cz} = 1.37/\Delta f )</td>
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</tbody>
</table>

* \( \lambda \) is the wavelength of the ultrasound center frequency; \( z_0 \) is the range to the focal zone; \( D \) is the diameter of the transducer aperture.
† \( \Delta f \) is the -6 dB bandwidth of the ultrasound pulse in MHz.

**II. AREA-WISE SIGNAL AND NOISE**

A tissue simulating phantom containing many scattering particles per resolution cell is insonified with a pulsed sinusoidal US beam from a focussed transducer which scans the phantom. We wish to consider the detectability of a disk-shaped signal embedded in the US speckle that appears in the B-Scan image. This is a classical problem in signal detection theory, a branch of statistical decision theory. The ultimate goal of decision theoretic analysis is to predict the performance of the decision maker, e.g., his true positive (hit) rate and false positive (false alarm) rate [12]. Usually the integrals for hit and false alarm rates are error functions of a summary physical parameter of the imaging system called the decision theoretic signal-to-noise ratio (SNR). This SNR uniquely specifies the performance of the ideal observer of the image. Our goal here will be to find the decision theoretic SNR appropriate to the task of disk (signal) detection in US B-Scan speckle (noise).

An optimal detector of disks in a noisy background will try to use all of the information over the area of the disk. One way to realize this is simply to average the signal, and thereby average the noise, over the disk area. We next calculate, therefore, the speckle signal and noise averaged over a finite area using the first- and second-order statistics of the speckle.

Although the magnitude of the complex pressure field received at the transducer is the quantity displayed in diagnostic US, we will see below that the intensity of that field plays a more fundamental role in the study of the performance of the ideal observer of US images. We will therefore first consider the signal and noise when intensity is averaged. The extension of the results for intensity to the magnitude case will be immediate. Our approach follows Goodman [13] and Wagner [14]. The signal obtained by averaging the intensity over a target area described by a weighting function \( A(x) \), where \( x \) refers to one- or two-dimensional variables, is

\[
I_A = \frac{1}{S} \int_{-\infty}^{\infty} A(x) I(x) dx,
\]

where

\[
S = \int_{-\infty}^{\infty} A(x) dx.
\]

For a uniform target or lesion area, i.e., \( A(x) \) equal to a constant value over a specified area, \( S \) will simply be the target area. We find for the mean value of \( I_A \), or ensemble average, \( \langle \cdot \rangle \),

\[
\langle I_A \rangle = \frac{1}{S} \int_{-\infty}^{\infty} A(x) \langle I(x) \rangle dx,
\]

\[
= \langle I \rangle,
\]

when \( \langle I(x) \rangle \equiv \langle I \rangle \) is not a function of position. As usual the variance of \( I_A \) is found as

\[
\sigma_A^2 = \langle I_A^2 \rangle - \langle I_A \rangle^2.
\]
We require therefore
\[
(I^2) = \frac{1}{S^2} \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 A(x_1) A(x_2) \langle I(x_1) I(x_2) \rangle,
\]
where the ensemble average in the integrand is recognized as the autocorrelation function of intensity, \( R_f(x_1, x_2) \) (see [11, eqs. (5)-(8)]). When the latter depends only on the coordinate difference, \( \Delta x = x_2 - x_1 \), we may write
\[
(I^2) = \frac{1}{S^2} \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 A(x_1) A(x_2) R_f(x_2 - x_1)
= \frac{1}{S^2} \left[ A(-\Delta x) \otimes R_f(\Delta x) \otimes A(\Delta x) \right]_{\Delta x = 0}
\]
a double convolution with the resulting variable \( \Delta x = 0 \). That is, we have simply the linear systems result [15], analogous to [11, eq. (7)] evaluated at the origin.

The autocorrelation function \( R_f(\Delta x) \) can be expressed in terms of the autocovariance function \( C_f(\Delta x) \) (see [11, eq. (7)])
\[
R_f(\Delta x) = C_f(\Delta x) + \langle I \rangle^2,
\]
Therefore, we obtain
\[
\sigma_I^2 = \frac{1}{S^2} \left[ A(\Delta x) \otimes C_f(\Delta x) \otimes A(\Delta x) \right]_{\Delta x = 0},
\]
since
\[
\langle I \rangle^2 = \frac{1}{S^2} \left[ A(\Delta x) \otimes \langle I \rangle^2 \otimes A(\Delta x) \right]_{\Delta x = 0} = \langle I \rangle^2,
\]
and we have assumed a symmetric area function \( A \). In the frequency domain this is simply [15]
\[
\sigma_I^2 = \int_{-\infty}^{\infty} df_x W_f(f_x) |T_A(f_x)|^2,
\]
where the Wiener spectrum \( W_f \) is the Fourier transform of \( C_f \), and \( T_A \) is the transform of \( A(x)/S \).

In general, it is necessary to carry out the specified integrals to obtain \( \sigma_I^2 \). However, a useful approximation follows immediately from (9). If the area of the disk is much greater than the region where the autocovariance \( C_f \) is appreciable, then
\[
A(\Delta x) \otimes C_f(\Delta x) \approx A(\Delta x) \left[ \int_{-\infty}^{\infty} d(\Delta x) C_f(\Delta x) \right]
= A(\Delta x) \left[ \int_{-\infty}^{\infty} d(\Delta x) C_f(\Delta x) / C_f(0) \right] C_f(0).
\]
That is, the convolution does not alter the shape of \( A \), it merely rescales it. The integral in brackets is a length (area) in one-(two-) dimensions and is called the correlation cell \( S_c \) of the speckle pattern [13]:
\[
S_c = \int_{-\infty}^{\infty} d(\Delta x) C_f(\Delta x) / C_f(0).
\]
The correlation cell was evaluated in the previous article [11] from the ultrasound speckle autocovariance function. In the lateral direction the size of the correlation cell \( S_c \approx 0.9 \lambda e_0 / \Delta f \) for both intensity and magnitude (see Table I-B). In the range direction the size of the cell was found to be \( S_c \approx 1.4 / \Delta f \) in millimeters where \( \Delta f \) is the full-width half-maximum transducer bandwidth in MHz. We may then write
\[
\sigma_I^2 = \frac{S_c}{S_m} \cdot C_f(0) = C_f(0)/M,
\]
where \( M \) is then the number of speckle correlation cells within the sample (measurement) area; in this case the sample area is a disk. Note that the concept of the cell size is equivalent to making the speckle correlation equal to unity over the cell and zero otherwise. Similarly, we would find the variance of the measurement of the averaged magnitude \( V \),
\[
\sigma_V^2 = C_v(0)/M.
\]
We may now use (4), (15), and (16) to write the following estimates for the SNR when averaging over a disk target area \( S_m \) that is large with respect to the speckle cell \( S_c \):
\[
\text{SNR}_{IA} = \frac{\langle I \rangle_1}{\sigma_I} = \frac{\langle I \rangle M_I^{1/2}}{C_f(0)^{1/2}} = M_I^{1/2}
\]
for averaged intensities, since \( C_f(0) = \langle I \rangle \) from [11, eq. (30)] and
\[
\text{SNR}_{VA} = \frac{\langle V \rangle_1}{\sigma_V} = \frac{\langle V \rangle M_V^{1/2}}{C_f(0)^{1/2}} = 1.91 M_V^{1/2}
\]
for averaged magnitudes, since \( C_V(0) = \langle V \rangle \) at a point in an ultrasound speckle image [11].

These SNR's refer to the estimate of a signal averaged over an area. Suppose the task is to distinguish a signal in a given area from the average background over a similar area. This is related to the statistical problem of estimating the difference between two noisy levels: the new signal of interest is level 1 (in signal area) minus level 2 (in background area); the new error or variance is the sum of the variances at level 1 and at level 2. That is, for intensity say,
\[
\langle \Delta I \rangle = I_1 - I_2
\]
\[
\sigma_{\Delta I}^2 = (I_1^2 + I_2^2)/M_I
\]
yielding the SNR for the difference signal
\[
\text{SNR}_{IA} = \frac{I_1 - I_2}{(I_1^2 + I_2^2)^{1/2}} \cdot M_I^{1/2}.
\]
Similarly, for magnitudes we have
\[
\text{SNR}_{\Delta V} = \frac{V_1 - V_2}{(V_1^2 + V_2^2)^{1/2}} (1.91) M_1^{1/2}.
\] (22)

For the low contrast limit, where \( I_1 \approx I_2 = I \), \( V_1 \approx V_2 = V \), we find
\[
\text{SNR}_{\Delta I} = \frac{\Delta I}{\sqrt{2} I} \cdot M_1^{1/2} = \frac{\Delta V}{V} M_1^{1/2}
\] (23)
and
\[
\text{SNR}_{\Delta V} = \frac{1.91}{\sqrt{2}} \frac{\Delta V}{V} M_1^{1/2},
\] (24)
where we have used
\[
 I = V^2
\]

and
\[
\Delta I = 2V(\Delta V).
\] (25)

It is tempting at this point to deduce the sought after contrast/detail function from the SNR's of (21) and (22). We will show in the next section that only one of these is fundamentally related to the performance of a lesion detection task. We will therefore postpone discussion of the SNR's of this section until that point.

### III. Statistical Decision Theory for US B-Scans

We next calculate from first principles the performance of the optimal observer/decision maker that uses the peak or envelope detected signal to perform a given signal detection task. The result will yield the optimal, or best performance, contrast/detail function. This will be accomplished by constructing a decision function to be used by an ideal observer of a B-mode image who must decide on the presence or absence of a given signal. It is shown in texts on signal detection theory [12] that using as a decision function the odds embodied in the likelihood ratio (defined below), or a quantity monotonic with the likelihood ratio, is equivalent to a number of "best performance" criteria. That is, we are able to calculate the best performance possible for detecting a signal of specified characteristics; this level of performance then becomes a goal or standard for the development of actual hardware, detectors, and displays for human observer applications. We limit our discussion of "performance" here to a calculation of the likelihood ratio or odds for the set of cell readings \( \{V_i\} \) from the following probabilities. First, using the Rayleigh pdf characteristic of the B-mode image, the likelihood or probability of the readings \( \{V_i\} \) given the presence of the signal level \( \psi_1 \) is
\[
L(\{V_i\} | \psi_1) = \prod_{i=1}^M (V_i/\psi_1) \exp(-V_i^2/2\psi_1).
\] (26)

The likelihood or probability of the readings given the presence of only the background level \( \psi_2 \) is
\[
L(\{V_i\} | \psi_2) = \prod_{i=1}^M (V_i/\psi_2) \exp(-V_i^2/2\psi_2).
\] (27)

The likelihood ratio for the readings is then
\[
\gamma_{12} = \frac{L(\{V_i\} | \psi_1)}{L(\{V_i\} | \psi_2)} = \prod_{i=1}^M \frac{\psi_2}{\psi_1} \exp \left[ \frac{V_i^2}{2} \left( \frac{1}{\psi_2} - \frac{1}{\psi_1} \right) \right].
\] (28)

We now define a quantity monotonic with the log likelihood ratio and therefore monotonic with the likelihood ratio
\[
\gamma_{12} = \log \left[ \left( \frac{\psi_1}{\psi_2} \right)^M \gamma_{12} \right] = \sum_{i=1}^M V_i^2 \left( \frac{\psi_1 - \psi_2}{2\psi_1\psi_2} \right).
\] (29)

The factors involving the \( \psi \) are constants related to the mean-square scattering strengths of regions 1 and 2, and are therefore independent of the measured values of the random variables \( V_i \), so it is sufficient to study
\[
\gamma_{12} = \sum_{i=1}^M V_i^2, \quad 1 \leq i \leq M,
\] (30)
as our decision function. The decision maker will respond that the disk is either present or absent depending on whether the value of \( \gamma_{12} \) is greater or smaller than some criterion or cut-off value. This is called a decision rule.

We now gain some insight concerning the value of \( M \). The ideal decision function for magnitude images involves squaring and summing the readings over the cells. That is, it is a prescription for using the intensity values. There are \( M \) such independent values. Therefore, as we study the statistics of the decision function it is \( M \), not \( M_V \), which is more funda-
mental. Henceforth $M$ will refer to $M_I$, since we will be dealing with squared magnitude, i.e., intensity values. The slight ambiguity about the number of cells arises from the fact that we are actually making an approximation to an exact solution of the problem. The exact solution requires finding a representation in which the noise is uncorrelated (i.e., an eigenvalue problem). Goodman [13] has shown that working with the correlation cell concept for intensity yields a very good approximation to the exact solution.

We now require the distribution for $\gamma_{12}$ to determine the performance of the decision maker. Since each $V_i$ comes from a Rayleigh distribution, $V_i^2$ will come from an exponential distribution, $p(V_i^2) = 1/(2 \psi) \exp(-V_i^2/2\psi)$, $V_i^2 > 0$ (corresponding to the intensity in the laser case, as shown in [11, eqs. (2)-(4)]). The sum over $i$ will tend to a Gaussian distribution, as $i$ goes to infinity, according to the central limit theorem. From [13, fig. 2.13] we see that for a value of $M$ of about 10 the Gaussian approximation becomes reasonable in practice. We require then only the mean of $\gamma_{12}$ given that the target is present, $E[\gamma_{12} | \psi_1]$, the mean of $\gamma_{12}$ given that the target is not present, $E[\gamma_{12} | \psi_2]$, and the corresponding variances. Using the results in Table I for each term in (29) these are found to be

$$E[\gamma_{12} | \psi_1] = 2M\psi_1; \quad E[\gamma_{12} | \psi_2] = 2M\psi_2$$
$$\text{var}[\gamma_{12} | \psi_1] = 4M\psi_1^2; \quad \text{var}[\gamma_{12} | \psi_2] = 4M\psi_2^2.$$  

The summary measure of the performance of this decision maker that determines hit and error rates is the ideal signal-to-noise ratio $\text{SNR}_{\text{opt}} = \Delta \mu / \sigma$, where

$$\Delta \mu = E[\gamma_{12} | \psi_1] - E[\gamma_{12} | \psi_2] \equiv 2M(\psi_1 - \psi_2)$$
$$\sigma = \sqrt{\text{var}[\gamma_{12} | \psi_1] + \text{var}[\gamma_{12} | \psi_2]} \equiv 2M^{1/2}[\psi_1^2 + \psi_2^2]^{1/2}$$

or

$$\text{SNR}_{\text{opt}} = M^{1/2} \frac{[\psi_1^2 - \psi_2^2]}{[\psi_1^2 + \psi_2^2]^{1/2}} = M^{1/2}C_\psi = \left(\frac{S}{S_c}\right)^{1/2}C_\psi.$$  

The quantity $C_\psi$ defined by this equation will be found to function as a contrast for small signal differences, but we should remember that it is fundamentally a signal-to-noise ratio. We may now find the ideal contrast/detail function for the detection of a disk lesion of diameter $d$, and "contrast" $C_\psi$ by setting $\text{SNR}_{\text{opt}}$ equal to some designated threshold value $\text{SNR}_T$:

$$\text{SNR}_T = \left[\frac{\pi d^2}{4} \frac{S_{\text{ex}}}{S_c} \right]^{1/2} \cdot C_\psi$$

and

$$C_\psi = \text{SNR}_T(S_{\text{ex}} \cdot S_c)^{1/2},$$

where we have assumed that the speckle cell shape is elliptical. This translates into an ideal contrast/detail or contrast-diameter curve with a slope -1 in log-log coordinates. That is, for a fixed level of ideal observer performance characterized by the value $\text{SNR}_{\text{opt}} = \text{SNR}_T$, a plot of log threshold "contrast" versus log threshold diameter has slope -1. The precise "hit" and "error" rates depend on the cut-off or criterion level chosen for the decision rule. These details are neglected in the present analysis.

If we repeat the above exercise for a detector that senses intensity, $I = V^2$, as in the laser case, the exponential distribution for $I_i$ is required,

$$p(I) = \frac{1}{2\sigma^2} \exp\left(-I/2\sigma^2\right), \quad I \geq 0,$$

or

$$= 0, \quad \text{otherwise.} \quad (35)$$

We find in this case

$$\text{SNR}_{\text{opt}} = M^{1/2} \frac{I_1 - I_2}{(I_1^2 + I_2^2)^{1/2}},$$

which is formally identical to (33), since $\psi \propto I$. This could have been anticipated from a general result of signal detection theory: a transformation from a decision variable $X$ to a decision variable $Y$ by a one-to-one relation e.g., $I = V^2$, does not change detection performance [17].

The results in (33) and (36) are identical to the result of (21). The process of working through the statistical decision theory for the optimal performance using a peak-detected or magnitude image has brought us back to the fundamental SNR for intensities. It is a description of the best possible performance using a magnitude (or intensity) image: the best possible performance is achieved by integrating $V^2$ or $I$ over the lesion and comparing this to the integrated value over a background of the same size. No statement is made here with regard to human performance.

The SNR of (22) for magnitudes was seen to have the same low contrast behavior as (21), i.e., the SNR of this section. That is, the heuristic approach of the previous section and the decision theoretic approach here agree for low contrasts. However, in the limit of high contrasts ($I_2 \to 0$) the magnitude SNR of (22) exceeds the intensity SNR by the ratio 1.91:1,00, as was the case for the point SNR's. The magnitude SNR cannot, therefore, be a true decision theoretic or best performance SNR, since (33) or (36) describes best performance. Otherwise it would suggest that improvements without limit would result from successive square roots of the B-scan image. The magnitude SNR's bearing the factor 1.91 cannot be used alone to calculate a fundamental measure of signal detectability, but would be useful if the true probability distribution function for the integrated magnitude were known. (For further discussion of this fundamental point, see [16].) Once the phase information is lost from a signal there is no fundamental advantage to using magnitudes instead of intensities, since they are easily retrievable from each other by an ideal detector.

The result of George et al. [6] is only an approximation to the rigorous result given here. The former result ignores the dependence of the speckle noise on the signal level $\psi$, and is therefore only exact in the limit of small contrasts. That is, the earlier result ignores the multiplicative nature of the noise [18].
IV. SOME HUMAN OBSERVER RESULTS

Having derived a framework in which to analyze contrast/detail performance, we now return to the measurements by Smith and Lopez [7] of the threshold detectability of simulated lesions in a tissue simulating phantom and the resulting dependence of threshold contrast (defined as $V_2/V_1$ in decibels) on lesion diameter for human observers. In Fig. 4(a) we have replotted the contrast/detail data from Fig. 3(a) in terms of $C_\psi$ versus diameter. Similarly in Fig. 4(b) the experimental contrast/detail data from Fig. 3(b) are plotted in terms of $C_\psi$ versus diameter. The results are seen to be reasonably well fit in log-log coordinates with a single straight line from a weighted regression analysis. The regression slopes and correlation coefficients for Figs. 4(a) and 4(b) are shown in the figure inserts.

We may compare the results for the slopes with the predicted slope of $-1$ for an ideal observer. For a given high contrast performance, the more negative the slope, the greater the low contrast sensitivity. The slopes of the experimental curves, $-0.86$ and $-0.75$, thus indicate that the performance of the human observer is less than ideal. Analogous data have been measured for x-ray noise (see Appendix), i.e., the slopes for the performance of human observers are equal to or less negative than the slope for the performance of the ideal observer.

Because the contrast $C_\psi$ is limited to the range of 0 to 1, the $y$-intercepts of the regression lines are nonphysical points. However, one can estimate the decision theoretic SNR$_T$ for the high contrast value $C_\psi = 1$. From (34) one notes that at $C_\psi = 1$. 

Fig. 4. (a) Contrast/detail data from Fig. 3(a) replotted in terms of $C_\psi$ (33) versus diameter. Error bars are ±1 standard deviation. (b) Contrast/detail data from Fig. 3(b) replotted in terms of $C_\psi$ versus diameter.
therefore portable threshold criteria; statistical tests for the
There is a need now to develop: consensus among investigators
ment gave a value of SNRT
i.e., point and edge blurring.
we speculate that the range of values is due more to the
limit [20]. We currently have programs underway in these
lesion, or M, takes on values less than
performance on the absolute scale determined by the ideal
difference of two contrast/detail curves; and observer effi-
V. SUMMARY DISCUSSION
Having arrived at an understanding of the detectability of focal lesions in ultrasound images, we should examine what technological developments might improve the performance of ultrasound scanning systems. The slopes of the experimental data, -0.75 and -0.86, indicate that some image processing techniques might be helpful in achieving the slightly more negative slope of -1, characteristic of the ideal observer. Smoothing techniques might be examined in this regard. Since this would degrade resolution limited detail, separate display for high and low contrast information might be necessary.
Detection is also enhanced by reducing the parameter (S_{ex} \cdot S_{cz}) of (34), i.e., reducing the correlation cell size to lower the y-intercept of Figs. 4(a) and 4(b). S_{ex} is essentially a measure of system resolution in the lateral direction. Therefore, use of higher frequency transducers to improve the two-point lateral resolution of the scanner should also improve detection of large low contrast lesions, as long as bandwidth and system sensitivity are maintained, and relative object contrast does not change. This shrinking of the speckle cell permits more independent samples of the target lesion to be taken. It must be accomplished at the transducer during the complex summation. Any nonlinear postprocessing, such as an nth root detector that reduces the speckle contrast, would be merely cosmetic. We saw earlier that working with the magnitude, or the square root of intensity, instead of intensity yields a lower speckle contrast but does not affect the intrinsic detectability of low contrast lesions, at least for the ideal observer. Cosmetic approaches may offer slight gains for real observers.
In choosing a higher ultrasound frequency to increase sampling of a lesion, one must recall that backscatter, a major source of lesion contrast, is a function of the scatterer size compared to the ultrasound wavelength [21]. This function ranges through an f^4 dependence to frequency independence (specular targets). Thus the object contrast of a low contrast target against a given background may also be frequency dependent.
Spatial compounding improves the detection of low contrast lesions by superimposing uncorrelated ultrasound images of the same target [22]. This technique can be treated immediately within the context of our analysis. The number of independent samples of a lesion, M in (33), increases linearly with the number of independent superimposed compound images. Therefore, (33) for signal-to-noise ratio, and (34) for the contrast/detail relation can be modified to become
\[
SNR_{opt} = C_{\psi} M^{1/2} N^{\frac{1}{2}}
\]
and
\[
C_{\psi} d = \frac{SNR_{T}(S_{ex} \cdot S_{cz})^{1/2}}{N^{1/2}},
\]
where N is the number of independent images compounded. Burckhardt [22] has shown that when spatial compounding by interrogating the target at different angles, the transducer must be moved a distance comparable to the aperture size to achieve uncorrelated data from the same target volume. In practice, then, spatial compounding will also increase image acquisition time.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>DETECTABILITY PARAMETERS FOR TWO SCANNERS</th>
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<tbody>
<tr>
<td></td>
<td>d (mm)</td>
</tr>
<tr>
<td>ADR Model 2130</td>
<td>@ C_{\psi} = 1.0</td>
</tr>
<tr>
<td>Unirad GZD</td>
<td>13 mm, 3.5 MHz Transducer</td>
</tr>
</tbody>
</table>

\[
d = SNR_{T}(S_{ex} \cdot S_{cz})^{1/2}.
\]
Axial resolution and detectability are simultaneously improved through reducing the parameter $S_{\text{res}}$ by expanding the transducer bandwidth. Broad-band transducers can also be used for frequency compounding [23]. In this case a given transducer bandwidth is broken down into several narrower band components to obtain separate uncorrelated images which are superimposed, thus increasing the independent sampling of the target. Magnin [24] has discussed measurements of the requirements for uncorrelated images when frequency compounding.

There are, thus, at least four possible techniques for improving the detectability of low contrast lesions:

1) higher transducer frequencies;
2) spatial compounding;
3) broader bandwidth transducers;
4) frequency compounding; and perhaps
5) image smoothing.

In comparing the relative effectiveness of these methods, one must consider the possible changes in object contrast, and the trade-offs with patient exposure and image acquisition time. These will serve as additional dimensions, beyond those of the present paper, on which one may carry out a complete optimization of ultrasonic imaging systems for the detection of focal lesions in parenchymal tissue.

APPENDIX: POISSON IMAGES REVISITED

In photon imaging, such as photography or radiography, the image statistics are characterized by the Poisson distribution—a one parameter distribution for which the variance $\sigma^2$ in the number of photons counted is equal to the expected number $\mu$. Suppose we have to detect a difference between the area density $N_1$ of detected particles over a disk of area $A$, and the density $N_2$ in similar areas in a uniform background. The total available signal will be the difference in the expected number of photons in each area, which is $(N_1 - N_2)A$. The noise will be the square root of the variance of this quantity which will be $(N_1 + N_2)^{1/2}A^{1/2}$. Then the signal-to-noise ratio $\text{SNR}_p$ is

$$\text{SNR}_p = \frac{(N_1 - N_2)A}{(N_1 + N_2)^{1/2}A^{1/2}}.$$  (A1)

As in all SNR analysis, the signals of interest subtract and the noise variances add. Note the symmetry with the US results of (33) and (46). In the limit of a small signal difference $N_1 \approx N_2 = N$, we have

$$\text{SNR}_p = \frac{\Delta N}{N} \left(\frac{(NA)^{1/2}}{2^{1/2}}\right), \quad \Delta N = N_1 - N_2,$$

$$= C(NA)^{1/2}/2^{1/2},$$  (A2)

where we have defined the contrast $C = \Delta N/N$. It can be shown that the results in (A1) and (A2) are rigorous for ideal observers (See [25] for general expression and references).

Also, the low contrast result is a rather well-known form that is variously referred to as the Rose model, Rose–Schade model, or matched filter [5], [14]. The popular Rose version of this expression neglects the $2^{1/2}$ in the denominator. Rose and Schade and others have found that this SNR (without the $2^{1/2}$) must take a value between 2 and 5 for human observers to detect a disk signal.

The expression in (A2) can be recast as

$$N^{1/2} C_d = \text{const} = \text{SNR}_T = k, \quad 2 \leq k \leq 5,$$  (A3)

where we define $\text{SNR}_T$, the threshold SNR, or $k$ in the Rose version, and find that there is an inverse relation between required contrast and lesion diameter $d \propto A^{1/2}$ at threshold, at least for ideal observers, at a fixed radiation level $N$. That is, the contrast-diameter curve in log-log coordinates for ideal observers has a slope of -1. Some human observer data from the work of Burgess et al. [19] with x-ray noise is presented in Fig. 5. The slope -2 portion of the curves is in the resolution limited regime which we do not discuss here. The data in the large diameter region fall on a curve with a slope that departs from the ideal as the diameter increases. The situation here strongly parallels the US case discussed in the body of the paper.

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The mention of commercial products, their source, or their use in connection with material reported herein is not to be construed as either an actual or implied endorsement of such products by the Department of Health and Human Services.

REFERENCES


[17] J. P. Egan, Signal Detection Theory and ROC Analysis. New York: Academic, 1975, append. B. (This point was brought to our attention through a derivation by David G. Brown of NCDRH of the invariance of the likelihood function under a change of variable that is one-to-one.)


