Plate Mode Coupling in Acoustic Surface Wave Devices

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Abstract—The acoustic modes of crystalline parallelopipeds are examined for their influence on the terminal properties of acoustic surface wave (Rayleigh mode) filters. It is shown that in substrates with parallel top and bottom surfaces, the entirety of the spurious acoustic responses can be described by the eigenmodes of the plate. Signal transmission between transducers on a plate and transducers on a half space is considered and modeling differences between using bulk waves, plate waves, or pseudosurface waves for the basis modes are discussed. Quartz, lithium niobate, bismuth germanium oxide, and bismuth silicon oxide are analyzed. It is concluded that cuts of lithium niobate taken from the range between about 30° (cubic) LiNbO₃ and 60° (cubic) LiNbO₃ can provide substantial advantages over more conventional cuts such as (50) LiNbO₃.

I. INTRODUCTION

Acoustic surface wave filters [1], [2] achieve band-pass shaping by spatially weighting the energy coupled into the crystal. This weighting can be accomplished by varying the amplitude or phase of the integrated energy radiated into an acoustic beam. In practice, these procedures are now developed to a high degree of perfection allowing the independent specification of both phase and amplitude response over the entire passband of the filter; amplitude error specifications of ±0.5 dB are common with adjacent channel and trap rejections of −60 dB being currently obtainable.

However, these high performance specifications cannot be met for all carrier frequencies and any substrate crystal. In addition to the obvious limitations set by the crystal parameters, such as radiation Q, propagation attenuation, and surface wave velocity, is the limitation of interference effects caused by other modes of energy transport in the crystal. All of the weighting techniques known for achieving a specified band-pass characteristic assume that the transduction process involves only one propagating mode. If other modes are capable of being excited and these modes are allowed to interact with some degree of phase coherence with the output transducer, then the terminal response to the filter will be degraded. Since the energy available to the other modes of an acoustic surface wave filter is usually small, the distortion to the main passband is also small; the most significant degradation occurs in trap depths and in the high frequency adjacent channel rejection where it is desirable to have no signal transmission at all.

We examine, here, the nature of the spurious modes of crystals commonly used for acoustic surface wave (Rayleigh wave) devices. The materials considered are listed in Table I along with many of their properties.

Figure 1 shows the geometry under consideration. X-axis propagation of the waves is assumed and the normal to the substrate is taken along the +z axis. The crystallographic coordinates of the crystal can have any orientation with respect to the space frame. Parallelism of the top and bottom surfaces of the substrate shown in Fig. 1 will be assumed in the mathematical development. Whether we consider the case illustrated, which is a plate, or assume a crystal with only one surface (i.e., a half space) is crucial to the type of modeling carried out and indeed to the answers obtained.

Bulk waves [3] [14], leaky waves or pseudosurface waves [15] [29], and plate waves [30] [31], [51] are terms used to describe the spurious modes in Rayleigh wave devices. Which model of the spurious mode process should be used to predict device performance is determined by the geometry of the device. For example, if the device consists of two broad-band transducers relatively far apart on a very thick crystal, then the bulk wave model of energy radiated into the bulk along the dashed line in Fig. 1 might be the best model. However, one would need to bear in mind that specular reflection from the bottom surface probably does not occur. Also if the crystal orientation is one which can have a pseudosurface wave, then a substantial part of the spurious acoustic energy exciting the output transducer may be omitted using the bulk mode model.

If the bottom surface of the crystal were modified by tilting it [35] [37], roughening it [38] [39], or effectively moving it to infinity [25] in order to eliminate reflected bulk mode energy from reaching the output transducer, then the problem to be solved would be transmission of a leaky or pseudosurface wave between transducers. In this case, all of the spurious energy reaching the...
TABLE I

Data for Materials Considered*

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>VELOCITY</th>
<th>(\Delta v/v)</th>
<th>MFR %</th>
<th>(C_{TD})</th>
<th>(a_2)</th>
<th>(a_4)</th>
<th>(a_6)</th>
<th>(a_8)</th>
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<td>0.0278</td>
<td>2.0</td>
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* Crystal orientations are defined according to the IEEE Standards 1521. The velocity difference between the first allowed plate mode and the Rayleigh mode is expressed as a percentage of the Rayleigh velocity in the column headed MFR (mode free range). \(C_{TD}\) is the temperature coefficient of time delay for the Rayleigh mode. A complete discussion of the parameters is given in Appendix A.

\[
v(\theta) = v_0 \left(1 + a_2 \theta^2 + a_4 \theta^4 + a_6 \theta^6 + a_8 \theta^8\right)
\]

Fig. 2. Inverse velocity curves for the (zx) plane of LiNbO\(_3\).

output transducer does so by traveling directly along the free upper surface. The modes which can do this are called leaky or pseudosurface waves because, in general, they are not bound to the free surface, but are attenuated along the propagation direction due to energy radiation into a shear wave which carries energy into the bulk of the crystal.

The final model of the spurious mode process, the plate mode model, can only be applied to devices with parallel top and bottom surfaces. In this case, all of the devices' spurious modes are plate modes; with the addition of a bottom surface, the pseudosurface wave of the half space and the continuum of bulk mode reflections become families of plate modes. If the thickness of the substrate is a small number of wavelengths (say 10) and the passband of the filter is narrow, then the plate mode model is even more appropriate for describing the device performance.

We will be using the plate mode model of acoustic energy transmission in the crystal. The rationale for choosing this model over the others is as follows. All surface wave devices are built on finite crystals and most often the crystals come from wafers sawed from a boule; the top and bottom surfaces of the wafer are parallel and consequently the mode spectrum of the crystal is essentially that of a plate.

To make the top and bottom surfaces of the crystal nonparallel or to add treatment to the bottom surface to make it a perfect acoustic absorber or diffuser are commercially unattractive options. The viability of the device in a commercial context is greatly improved if it can be treated by standard semiconductor processing which permits batch fabrication and does not require labor-intensive special effort treatments. Consequently, there is strong incentive to have the substrate configuration be that of a plate. Analyzing the device performance in terms of lossless plate modes gives a worst case representation of the filter properties.

In Section II-A, we will develop the plate mode boundary value problem in detail showing the excellent agreement with experiment obtainable from the model. Here we will consider the physical processes involved in acoustic energy propagation both for the crystalline plate and the crystalline half space.

Four acoustic wave solutions most people are familiar with are the Rayleigh wave, bound to the surface of a half space, and the three plane wave bulk mode solutions of an infinite space. Figure 2 shows the inverse velocity...
plots of these four waves for a LiNbO₃ crystal cut with the (x, z) crystallographic plane in the sagittal plane of the crystal. For illustration we consider the case where \( \theta = 0 \) giving the crystallographic \( z \) and \( x \) axes coincident with the space coordinates of Fig. 1. Under these conditions, for propagation along the \( x \) axis, one finds that the Rayleigh mode is slowest; going up in velocity about 8%, the curves indicate the existence of a propagating "slow" shear mode; higher still in velocity, a "fast" shear wave at 124% of the Rayleigh wave velocity is found, and finally at 174% of the Rayleigh velocity, a propagating longitudinal mode is found. As mentioned above, these bulk modes exist in an infinite medium and are shown in Fig. 2 without reference to the fact that they may not be purely propagating along a free crystal surface.

For propagation in a half space downward away from the free surface of the crystal, one feels relatively certain that the bulk modes of an infinite medium accurately describe all the possible sources of acoustic energy transmission. Reference to Fig. 2 with \( \theta = 0 \) shows that for bulk wave propagation downward from the free surface of the crystal with an \( x \)-component of velocity greater than the on-axis slow shear wave velocity, there is always some mode which can carry energy away from the surface. We see therefore that if the surface of a half space is excited periodically—as in the inset in Fig. 1—at a frequency higher than that of synchronism to the slow shear wave, there can be continuous radiation into the bulk modes of the medium. The \( x \)-component of velocity is determined by the frequency and the periodicity of the exciting array, and the \( z \)-component of velocity is determined from the inverse velocity plots of the propagating bulk modes. These two components, of course, prescribe the angle of the radiated beam of energy.

In general, a bulk mode propagating away from a source on the surface of the crystal does not satisfy the free surface boundary conditions. Consequently, once away from the transduction region of the surface, the acoustic disturbance will not propagate along the surface. Excitation of bulk modes can certainly take place at a transducer where the boundary conditions are not free, but the energy thus coupled in will propagate down into the interior of the half space where there are not boundary compatibility problems. Consequently, transducers spaced adequately far apart on a half space do not interact through the bulk mode spectrum of the crystal since these modes do not in general satisfy the free surface boundary condition that exists between the transducers. However, the bulk mode excitation spectrum of each transducer is continuous as a function of frequency with the energy being carried away into the half space.

Direct spurious acoustic communication between the transducers on a half space can occur through leaky or pseudosurface waves. These waves which are composed of a sum of generalized bulk waves with one or more bulk waves carrying energy down into the half space can satisfy the free surface boundary conditions. In many cases, the energy loss of the wave is small and considerable spurious mode distortion of the filter transfer function is thus possible.

When there is another boundary parallel to the top surface, the problem is considerably different. Now there can be waves traveling upward in the plate as well as downward. Also boundary conditions at two surfaces must now be satisfied. The transducers on a surface of the plate can still excite disturbances continuously as a function of frequency since inhomogeneous boundary conditions exist at the surface covered by the transducer but the disturbance will satisfy free surface boundary conditions at the bottom of the plate beneath the transducers and at both top and bottom surfaces away from the transducer at only discrete velocities. At these discrete velocities the mode, a plate mode, will freely propagate along the plate. At other velocities, the disturbance is evanescent away from the transducer and does not carry energy away. To find the allowed modes of the plate, boundary conditions applicable to the acoustic disturbance must be imposed and the boundary condition matrix solved for singularities.

The solution of this problem is carried out in the next section. It is shown that solutions to the boundary value problem can be grouped into families, each associated with one branch of the bulk mode inverse velocity curves illustrated in Fig. 2. The first family of plate modes starts at about the velocity of the on-axis slow shear bulk wave. Additional families start at about the velocities of the on-axis fast shear bulk wave and longitudinal bulk wave.

The most troublesome of these modes from the point of view of the acoustic surface wave filter designer are those with velocities very near to that of the Rayleigh mode because they can carry energy at frequencies in the passband of the filter. Those modes farther out in velocity, say the longitudinal modes, are typically an octave away from the center of the passband and consequently are easily eliminated by tuning networks.

The near-in modes cannot be discriminated against by system transmission characteristics; they must be eliminated in the acoustic surface wave filter or be allowed to set device performance limits. Obviously, it becomes highly desirable to know what properties of the modes can be used to discriminate against their existence. Simply stated, the problem is to eliminate all forms of acoustic energy transmission from the input to the output transducer except the Rayleigh mode. In general, the problem subdivides into two different approaches: accept the excitation of other modes and intercept them before they excite the output port or seek crystal and system parameters which prohibit the excitation of other modes.

Following the first approach, one finds many techniques for discriminating against the excited plate modes [35]-[41]. However, nearly all of them require some enlargement of the substrate surface area. The use of multistrip couplers [41] and dual acoustic beams [40] are two such methods. In this work, we consider one acoustic beam only, which is as narrow as diffraction limits will permit, with only two transducers spaced apart with only as
much free propagation space as state-of-the-art packaging techniques require for cross-talk suppression to the $-90$-dB level. In short, we limit ourselves to consideration of the smallest possible substrate size. With that limitation, the first approach reduces to treatment of the back surface of the crystal. Techniques along those lines have been developed and are treated in References [35]-[39].

The second approach calling for crystal conditions in which the plate modes are not excited [28], [42], and the general theoretical basis for filter response distortion by plate modes will be treated in detail here. Our objectives are to first quantitatively define the problem of spurious mode distortion in parallelopipeds and then to seek cuts of commonly used crystals in which the plate modes of the crystal are either electrically uncoupled from the transducers or have velocities as large as possible with respect to the Rayleigh mode. In the latter case, by working at a sufficiently high carrier frequency, the spurious mode images appear out of band of the Rayleigh mode band-pass characteristic. In addition, in the search for a new crystal cut, we keep in mind that the cut must have acceptable properties in other respects. From a diffraction point of view, it would be desirable if the cut were a focusing direction with a parabolic velocity surface. Fractional bandwidth requirements dictate the need for high electromechanical coupling and general system utility requires a low coefficient of temperature sensitivity. Numerical data on these properties are given in Table I and the discussion of the properties is in Appendix A.

II. ANALYSIS MODEL

A. Acoustic Plate Mode Boundary Value Problem

As indicated above, the geometry under consideration is illustrated in Fig. 1. We take the $x$ spatial coordinate to be the propagation direction with the $z$ coordinate along the substrate normal. Parallelism of the top and bottom surfaces of the solid plate is assumed. The crystal axes may have any orientation with respect to the space coordinates and the analysis will allow for completely general linear anisotropy and piezoelectricity.

The method of analysis, the superposition of partial waves, calls for combining the equations of motion for the medium and obtaining a secular equation for plane wave propagation in an infinite system. The secular equation is then solved for all possible generalized bulk modes. The word generalized is emphasized because the modes need not be purely propagating but may be evanescent in some direction. In our case, we will be seeking propagation along the plate and will therefore assume $x$ propagation of the bulk waves but allow for a complex component of $k$ vector in the $z$ direction. Of course, we assume straight-crested waves with no variation along the $y$ axis. Having found all of the possible generalized bulk modes corresponding to a given $x$ component of velocity, a superposition of the modes is taken. This superposition is required to match the boundary conditions of the plate.

The equations governing the motion of the body are

Maxwell's equations, Newton's second law, the equation of continuity, and the constitutive relations for the material,

\begin{align*}
- \nabla \times E &= \mu \frac{\partial H}{\partial t} \\
\nabla \times H &= \frac{\partial D}{\partial t} \\
\nabla \cdot \mathbf{T} &= \rho \frac{\partial \mathbf{v}}{\partial t} \\
\nabla \cdot \mathbf{v} &= \frac{\partial \mathbf{S}}{\partial t} \\
D &= \epsilon_0 \cdot E + \sigma_s \cdot \mathbf{S}
\end{align*}

and

\begin{align*}
\mathbf{T} &= \epsilon_0 \cdot \mathbf{S} - \sigma_s \cdot \mathbf{E}
\end{align*}

Reduced subscript notation is used, and in cartesian coordinates, the tensor operators, $\nabla \cdot$ and $\nabla \times$, have the matrix representation

\begin{align*}
[\nabla \cdot] &= [\nabla \times]^T = \\
&= \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\
0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 \\
0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x}
\end{bmatrix}.
\end{align*}

If propagation as $\exp[-j\beta(x + l_z - (\omega/\beta)t)]$ is assumed, then the propagation vector can be written

\begin{align*}
k &= \beta \begin{bmatrix} 1 \\ 0 \\ l_z \end{bmatrix} = \beta l
\end{align*}

and the tensor operator $\nabla \cdot$ takes the form

\begin{align*}
[\nabla \cdot]_{ij} &= -j\beta T = -j\beta \begin{bmatrix} 1 & 0 & 0 & l_z & 0 \\ 0 & 0 & l_z & 0 & 1 \\ 0 & l_z & 0 & 1 & 0
\end{bmatrix}
\end{align*}

Equations 8 and 9 can be considered definitions of the vector $l$ and the tensor $\mathbf{T}$.

After invoking the electrostatic approximation, Eq. 1-4, 8, and 9 can be combined to yield the stiffened Christoffel equation \[43\]

\begin{align*}
\mathbf{T} \left( \frac{\epsilon_0}{\epsilon_0 - \sigma_s} \frac{1}{l_c} \right) \mathbf{v} - \frac{\sigma_s}{\epsilon_0 - \sigma_s} \mathbf{v} = 0
\end{align*}

where $\mathbf{u}$ is the particle displacement vector.

When the matrix representations for the vector and
tensors in Eq. 10 are substituted and the terms multiplied out, a 12th order equation in the scalar decay constant $\xi$ is obtained. The equation can be reduced to 8th order by algebraic manipulations which permit factoring out a 4th order polynomial due to Poisson’s equation

$$[\varepsilon_{11} \xi^2 + 2 \varepsilon_{12} \xi_3 + \varepsilon_{13}^2] = 0. \tag{11}$$

The resultant eighth order secular equation specifies the $z$ component of wave vector for all possible plane waves which can exist in the medium having an $x$ component of velocity, $v = \omega/\beta$, and no variation in the $y$ direction.

An illustration of the solutions obtained from Eq. 10 is given in Fig. 3. The secular equation was solved for ST-cut quartz at an $x$ velocity of 5400 m/sec. We see that four of the roots are pure real corresponding to pure bulk mode propagation. The other four are complex. Two of the four complex roots correspond to the bulk quasilongitudinal modes and become pure real for $x$ velocities larger than the on axis quasilongitudinal velocity. The other two roots correspond to solutions of Poisson’s equations and remain complex at velocities far below the speed of light.

The eight modes found from Eq. 10 must be superposed to obtain solutions to the plate mode boundary value problem. The boundary conditions which we have applied are stress-free top and bottom surfaces and either electrically shorted top and bottom surfaces, or a shorted bottom surface and continuity of fields through the top surface. A boundary value determinant corresponding to these eight conditions is written and examined for singularities as a function of $x$-axis velocity. One finds that at discrete velocities all of the boundary conditions can be satisfied. The superpositions of plane waves at these eigenvelocities are the eigenmodes or plate modes of the crystal.

The plate mode spectrum of an ST-cut quartz plate is illustrated in Fig. 4. A plate thickness of $152/2\pi$ wavelengths was assumed in the calculation. Each vertical line represents an allowed mode of the plate and the abscissa value gives the velocity of the mode relative to the Rayleigh velocity for the case of a shorted bottom surface and continuity of electric fields through the top surface. The ordinate gives a relative measure of coupling to the mode by an interdigital transducer. Shown is $20 \log (\Delta v/v)$ where $\Delta v$ is the velocity difference between a mode propagating with a free top surface and the same mode propagating with a shorted top surface. The coupling strengths are shown with respect to the Rayleigh mode coupling which is defined to be 0 dB.

The mode chart of Fig. 4 shows the family of slow quasilear modes that begins at a velocity about 4.5% above the Rayleigh velocity. At relative velocities of about 1.61 and 1.82, the fast quasilear and quasilongitudinal modes become propagating. The onset velocities of the latter two families are closely spaced as would be expected from the inverse velocity plots of Fig. 3. After a relative velocity of 1.82, the modes from all three families together give a density of modes approaching that of a continuum. Indeed, the plate mode energy excited and received by interdigital transducers operated in a frequency range corresponding to synchronism at those velocities shows a nearly continuous behavior with a magnitude following the envelope of the modes.

B. Impedance Properties of the Plate Modes

Before the terminal properties of the filter can be predicted from the normal mode spectra, their impedance characteristics must be known. To carry out this determination with complete rigor is a formidable task to say the least. The transducer structures used in state-of-the-art acoustic surface wave filters are highly complex and are built up by approximation techniques which challenge analytic expression. Commonly today, a surface wave filter will employ both a finger withdrawal transducer and an apodized transducer. Neither of these structures is readily representable in concise analytic forms. The finger withdrawal transducer is perhaps the more tractable of the two because it has only a single acoustic beamwidth. For transducers with a single acoustic beamwidth, Auld and Kino [44] developed a variational analysis of the
transducer impedance. They applied their work to standard broad-band transducers, but the formalism is easily extended to describe finger withdrawal transducers if the class of trial functions for the charge density set up by the electrodes is appropriately modified. In their analysis, the fields of the acoustic disturbance are expanded in terms of the normal modes of the crystal in much the same way that normal mode impedance calculations are carried out for electromagnetic waveguide apertures. The field quantities are then substituted into a power theorem to obtain the impedance. Their result for the impedance is

\[ Z = \frac{w}{jQ_T^2} \sum_{m=1}^{\infty} \frac{\beta_m \phi_m \phi_m^*}{2P_m} \sigma_T(\beta) \sigma_T(-\beta) d\beta \]  

(12)

where \( w \) is the width of the acoustic beam, \( Q_T \) is the total charge entering one pad of the transducer, \( \beta_m \) is the propagation constant of the \( m \)th mode of the crystal, \( P_m \) is the power flow per unit width of the \( m \)th mode, and \( \sigma_T(\beta) \) is the transform of the surface charge density set up on the surface of the crystal by the metal fingers. This expression, which gives the impedance variation with frequency, also gives the contribution of the finger withdrawal transducer to the overall filter transfer function since the acoustic beam is all of one width.

In the examination of \( z \), we start by separating the modes which are synchronous with a component of the Fourier spectrum of the electrodes from the rest of the modes. Following Auld and Kino, we assume that the other modes are sufficiently asynchronous that they are not excited. Consequently, their aggregate effect may be evaluated from electrostatics. When \( q \) modes are synchronous with the transducer, Eq. (12) becomes

\[ Z = \frac{w}{jQ_T^2} \sum_{m=1}^{q} \frac{\beta_m \phi_m \phi_m^*}{2P_m} + \frac{1}{j\omega C_T} \]  

(13)

where

\[ S(\beta_m) = \frac{1}{j} \int_{-\infty}^{\infty} \frac{\beta_m}{\beta^2 - \beta_m^2} \sigma_T(\beta) \sigma_T(-\beta) d\beta \]  

(14)

and \( 1/j\omega C_T \) equals the summation over the asynchronous modes. The wave impedance, \( Z_m \) (\( = \phi_m \phi_m^*/2P_m \)), appears above and is a fundamental property of the \( m \)th mode. It has a \( 1/\omega \) frequency dependence and is related to the often quoted coupling parameter \( (\Delta v/\nu)_m \). Kino and Reeder [45] showed that when the \( m \)th mode is appreciably separated from all other modes,

\[ Z_m = \frac{\phi_m \phi_m^*}{2P_m} = \frac{2}{\omega(\epsilon_p + \epsilon_v)} \left( \frac{\Delta v}{\nu} \right)_m \]  

(15)

where \( \epsilon_p = (\epsilon_{\nu_p} - \epsilon_p^2) \) and \( \nu_m \) is the velocity of the \( m \)th mode. Substituting this result above, we find that

\[ Z(\omega) = \frac{w}{Q_T^2} \sum_{m=1}^{q} \frac{2}{(\epsilon_p + \epsilon_v)} \frac{S(\omega/\nu_m)}{\omega} \left( \frac{\Delta v}{\nu} \right)_m + \frac{1}{j\omega C_T} \]  

(16)

This is the desired relation; it describes an equivalent circuit like that shown in Fig. 5. The terminals of the transducer present a resistance \( R_m \) corresponding to energy carried away by the \( m \)th acoustic mode, a reactance \( X_m \) corresponding to energy exchanged between the \( m \)th mode and the transducer fingers due to asynchronous wave propagation, and a reactance \( 1/j\omega C_T \) corresponding to the static capacitance of the interdigital metallization.

It is clear from Eq. (16) that for any mode of the crystal, the real part of the impedance will vary inversely with the frequency of excitation and proportionally with the frequency dependence and is related to the propagation constant of the wave, \( \beta_m \), and not the frequency, independent of what mode is considered. This result is a consequence of the transducer effecting spatial excitation and thus being fundamentally related to mode repetition in space rather than repetition in time.

The equivalent circuit representation of Fig. 5 for the \( m \)th mode begins to diverge from an accurate representation of the \( m \)th mode properties wherever the density of modes around the \( m \)th one becomes great or there is an extremely strongly coupled mode nearby. In Fig. 4, for example, where the mode couplings \( (\Delta v/\nu)_m \) of the plate of ST-cut quartz are shown, divergence of the impedance properties predicted in Eq. (16) for a mode at \( v/\nu_{\text{RAY}} = 1.001 \) would be expected because of the relatively strong Rayleigh mode adjacent to it at \( v/\nu_{\text{RAY}} = 1.0 \). Also for \( v/\nu_{\text{RAY}} > 1.0 \) the density of modes is sufficient that the predictions from Eq. (16) are certain to be in some error. Nevertheless, if one carries out the calculations for the transducer interaction with the \( m \)th mode, the agreement with experiment is still exceptionally good as will be demonstrated.

The above theoretical development of the impedance properties of the transducer was verified by application to a very narrow band filter which utilized an ST-cut quartz substrate. The equivalent circuit for the multimoded acoustic surface wave filter shown in Fig. 5(b) was considered to describe the entirety of the terminal properties of the transducer with each impedance element \( R_m + jX_m \) given by the first term of Eq. (16). Since the experiment was carried out on ST-cut quartz crystals, \( |Z_m| < 1/j\omega C_T \) for all \( m \), because of the weak coupling of the quartz.
Accordingly, the current into the circuit is determined by

$$\frac{1}{j\omega C_T}$$.

We can then write that the power into the $m^{th}$ mode of the crystal is

$$P_m = \omega K(\beta_m) \left( \frac{\Delta v}{v} \right)_m$$

(17)

where $\omega$ is the operating frequency of the applied voltage, $K(\beta_m)$ is the effective propagation vector selectivity of the transducer, and $(\Delta v/v)_m$ is the coupling for the $m^{th}$ mode. Were the input and output transducers identical, then by reciprocity the power into the output load would be the square of Eq. 17. In fact, for the device used in the experiment, the output transducer was apodized. In this case, it is convenient to think of the apodized transducer as divided into channels of unapodized broad-band transducers connected in parallel. For each channel, the propagation vector selectivity function $K$ would be different but fundamentally the transfer of power in or out of the crystal for each channel would exhibit a frequency dependence as in Eq. 17. The frequency dependence of the radiation resistance of all the channels in parallel would have an effective selectivity function $K_{AP}(\beta_m)$ but would remain inversely proportional to frequency and proportional to $(\Delta v/v)_m$. Thus the power out can be written

$$(P_m)_{\text{out}} = A\omega^2 K_{AP}(\beta_m) K_{FW}(\beta_m) \left( \frac{\Delta v}{v} \right)_m$$

(18)

where $A$ is a proportionality constant, $K_{FW}(\beta_m)$ is the finger withdrawal propagation vector selectivity function, and $K_{AP}(\beta_m)$ is the effective propagation vector selectivity function of the apodized transducer. The power dissipated in the terminating load is also given by

$$\frac{1}{2} V_{\text{out}}^2 / R_{\text{LOAD}}$$.

We find, therefore, that the magnitude of the output voltage is given by

$$|V_m|_{\text{out}} = R_{\text{LOAD}} \left[ K_{FW}(\beta_m) K_{AP}(\beta_m) \right]^{-1/2} \left( \frac{\Delta v}{v} \right)_m$$

(19)

In determining the phase of $V_m$, we assume that the transducers are nondispersive with the phase of the wave at the physical center of the transducer coincident with the phase of the applied voltage. Since the phase at the output port must include the phase progression of the mode along the crystal we obtain

$$V_m = |V_m|_{\text{out}} \exp \left[ -j\beta_m d \right]$$

(20)

where $d$ is the distance between transducers.

Finally, the exact numerical values for $P_{FW}$ and $K_{AP}$ must be evaluated. To obtain these functions analytically for the complex structures used in the experiment is prohibitively difficult. Therefore, their combined effect was measured at one mode, the Rayleigh mode, and stored in the numerical procedure used to evaluate the entire modal response of the filter.

As stated before, the filter examined was on ST-cut quartz and had a Rayleigh mode response at 30 MHz. The frequency response was highly selective being only 1 MHz wide at the $-65$-dB level. Experimental results for the filter are shown in the upper trace of Fig. 6. The crystal thickness was such that at midband of any mode $t/\lambda = 152/2\pi$; consequently, the mode spectrum of Fig. 4 shows all the possible modes of the filter.

The mode data of Fig. 4 and Eq. 20 and the experimental determination of $K_{FW}(\beta_m) K_{AP}(\beta_m)$ were used to calculate the output voltage of each of the first 96 modes of the crystal at all frequencies from 29 MHz to 57 MHz. A phasor sum of the output voltages was taken and the theoretical insertion loss thus determined. The lower curve in Fig. 6 shows the prediction. Agreement between experiment and theory is excellent across the band. The only significant departure from agreement that stands out is the absence of the zeros in transmission between modes in the 37 MHz to 41 MHz range. Experimentally those zeros are a consequence of phasing between two adjacent modes. Also it will be noted that the transmission surrounding those modes is narrowed somewhat. To include these effects theoretically would have meant including dispersion in the mode velocity description, that is, $\beta_m = \omega/v(\beta_m)$. The potential improvement in the analytic prediction is small, and we consequently made the assumption that for all $m$, $\beta_m$ was independent of propagation constant.
ment between experiment and theory is good. Considering
the limitations of the model in the dense mode range,
better agreement could not be expected.

III. PLATE MODES OF COMMERCIAL
CRYSTALS

The results of the preceding section show very clearly
the validity of describing the spurious responses of acoustic
surface wave filters in terms of the plate mode spectrum
of the substrate. The questions to be posed, and somwhat
answered, in this section are: What properties of the crystal
control the mode spectrum? What conditions should be
sought in order to eliminate or diminish the effect of the
spurious modes? And, finally, what are the tradeoffs in
seeking partial answers to these problems with crystal
orientations other than the currently accepted crystal
cuts?

First let us consider coupling. We know from the
preceding development that transducer coupling is re-
lated to wave impedance \( \phi_m \phi_m^*/2P_m \), and we can write
that for any mode, the electric potential is given by

\[
\phi_m = \sum_{n=1}^{N} C_n \left[ \varepsilon_{33} \varepsilon_{33}^* + (\varepsilon_{36} + \varepsilon_{12}) b_{3n} + (\varepsilon_{36} + \varepsilon_{13}) b_{3n} \\
+ \varepsilon_{33} \varepsilon_{33}^* + (\varepsilon_{16} + \varepsilon_{11}) b_{3n} + (\varepsilon_{14} + \varepsilon_{13}) b_{3n} \\
+ \varepsilon_{33} \varepsilon_{33}^* + (\varepsilon_{26} + \varepsilon_{14}) b_{3n} + (\varepsilon_{26} + \varepsilon_{13}) b_{3n} \right] \tag{21}
\]

where \( b_n \) is the particle displacement vector of the \( n \)th
(out of \( 8 \)) partial wave of the \( n \)th mode of the plate and
\( C_n \) is the complex coefficient for that partial wave. Eq-
uation 21 demonstrates the complexity of visualizing the
general coupling of an arbitrary mode to the transducer.
In the completely general case, it is virtually impossible
to predict orientations of the crystal where the complement
of piezoelectric stress coefficients \( \vec{e} \) will combine with the
eigenvector of the normal mode of the system to produce
\( \phi_m \approx 0 \). Indeed, it is not believed to be true that such a
possibility exists for all \( m \). Efforts to decouple families of
modes in certain crystals have been made. The procedure
followed in that work \[28\], and in our own investiga-
tion was to examine Equation 21 where the eigenvector was
that of a bulk wave for orientations that produced no
electric potential. If a substrate were then cut with its
propagation surface along the direction where coupling
to the on-axis bulk mode vanished, then the first coupled
spurious mode of the substrate would have its energy
directed toward the bottom surface at a reasonably steep
gle. Assuming that those modes directed toward the
bottom surface could be absorbed, without reflection to
the output transducer, then the spurious mode inter-
fERENCE in the crystal could be eliminated. The exa-
nination of Eq. 21 reveals only orientations where the first
mode, which has partial wave components propagating at
very shallow angles to the top surface, is decoupled.
The rest of the spurious mode spectrum must be removed at
the bottom surface of the substrate. One of the cuts
asserted \[28\] to have highly superior spurious mode
properties is (110)-cut, (001)-propagation Bismuth Silicon
Oxide. Calculations carried out here and presented later
in this report show that those modes after the first shear
plate mode are in fact strongly coupled to the transducer.
Their removal from the terminal response requires special
 treatment of the bottom surface of the substrate.

One commonly occurring special case of acoustic mode
family polarizations is that in which the quasiliogitudinal
family and one quasishear family are polarized in the
sagittal plane, and the other shear family is polarized
perpendicular to the sagittal plane. In such a case, Eq. 21
can be useful in visualizing how the shear family with
polarization perpendicular to the sagittal plane can be
uncoupled from the transducers. In this special case, Eq. 21
becomes

\[
\phi_m = \sum_{n=1}^{N} C_n \left[ \varepsilon_{33} \varepsilon_{33}^* + (\varepsilon_{36} + \varepsilon_{13}) b_{3n} + (\varepsilon_{26} + \varepsilon_{13}) b_{3n} \right] \tag{22}
\]

Now if the piezoelectric constants are such that \( e_{31} =
(\varepsilon_{36} + \varepsilon_{13}) = e_{15} = 0 \), then all the modes of the family are
uncoupled. This situation obtains in the \((yz)\) plane of
LiNbO\(_3\). It leads to some advantageous orientations which
we will present later in this section. It also leads to some
seemingly paradoxical results as we showed in Reference 51.

While finding conditions in which \( \phi_m \) vanishes will
certainly eliminate the coupling by causing \( \phi_m \phi_m^*/2P_m \)
to go to zero, another way to diminish the coupling is to
increase \( P_m \) for a given \( \phi_m \), thus decreasing the wave
impedance and \((\Delta v/v)_m\). An easy way to accomplish this
is to make the substrate thicker. In very simple terms, if
the mode fills the substrate and the substrate is made
thicker, then the total power carried by the mode for a
fixed particle displacement at the surface will increase.
This effect is illustrated very well by a comparison of
Fig. 7 with Fig. 4. Both figures are for \(ST\)-cut, \(x\)-propaga-
tion on quartz, but Fig. 4 is for a thickness of \( t/\lambda = 152/2\pi \),
while Fig. 7 is for a thickness of \( t/\lambda = 60/2\pi \). Coupling to
the shear wave family in the range \( 1.0 \leq v/v_{Rav} \leq 1.5 \) is
about 8 dB higher for the thinner crystal \((20 \log \frac{152}{60} \approx 8.1)\). There are considerably fewer modes, as
would be expected, in the thin crystal but nevertheless
the out-of-band rejection would be substantially worse in
filters built on the thinner crystal.

Fig. 7. Mode spectrum of an \(ST\)-cut quartz plate that is \(60/2\pi\)
wave-lengths thick.
In addition to the relative strength of coupling of a plate mode, another consideration of fundamental importance to the filter design is the proximity of the first coupled plate mode to the Rayleigh wave mode. Here we have a solid basis for saying where the families of modes will begin; each family starts at just above the velocity corresponding to the minimum on-axis component of velocity achieved on the bulk wave velocity surface. In many crystal cuts, the minimum on-axis velocity component is achieved for propagation directly along the plate axis. An example of such a case was illustrated earlier in the Introduction by reference to Fig. 2 for (xz) LiNbO$_3$ with $\theta = 0$. There are some cuts, however, where the minimum on-axis component of velocity occurs for wave propagation slightly down into the crystal. Crystal cuts where this occurs are (yz) LiNbO$_3$, (zy) LiNbO$_3$, and (xz) LiNbO$_3$. Data on the fractional velocity difference from the Rayleigh mode to the first allowed plate mode (whether the allowed plate mode is coupled or not) for the crystal cuts considered here are shown in Table 1 in the column headed MFR (mode free range).

For the (xz) orientation of LiNbO$_3$ shown in Fig. 2, the on-axis shear wave is about 8% faster than the Rayleigh mode. The on-axis fast shear wave is about 24% faster than the Rayleigh mode and the longitudinal wave is about 74% faster than the Rayleigh mode. Mode coupling for a plate of (xz) LiNbO$_3$ is shown in Fig. 8. The onset of the two shear wave families is clearly evident in the theoretical plot, and the relative position of the modes in the experimentally obtained inset leaves no doubt that from the Rayleigh mode to 8% above the Rayleigh mode, there are no allowed modes for (xz) LiNbO$_3$.

The two preceding discussions of plate mode coupling levels and relative positions of the families of modes have somewhat answered the first question posed in the introduction to this section. The second question of what conditions lead to the elimination or reduction in the effects of the plate modes is harder to answer. Others [28] have sought crystal conditions which are contingent on an ability to absorb or incoherently scatter all spurious energy from the bottom surface of the substrate. This is a commercially unattractive solution and our approach has been to emphasize the exploitation of intrinsic material properties. It is acknowledged that some degree of bottom surface treatment [39] of the substrate may be necessary no matter what the crystal orientation, and we make use of the procedures as necessary. But we seek to minimize the dependence on these techniques. One solution which we have pursued [42] is to seek orientations of crystal in which not only is the first plate mode uncoupled but the first coupled mode is significantly higher in velocity than the Rayleigh mode. At a sufficiently high IF frequency, then all the spurious modes are out of band and can additionally be discriminated against by the system. The way these orientations are found is by examining inverse velocity plots like those in Figs. 2 and 3 for the maximum distance between the Rayleigh mode and the tangent to the slow shear wave branch. In isotropic media, this distance is a constant, but in anisotropic crystals, the distance varies with orientation; the more anisotropic the crystal, the greater the possible distance.

A. Quartz

Quartz is an extremely anisotropic crystal as the plots of Fig. 3 illustrate. For the standard ST-cut $x$-propagation substrate, the slow shear wave is only 4.5% faster than the Rayleigh mode. However, for propagation on a crystal cut with its sagittal plane in the plane of Fig. 3, and having propagation 42.5° above the $x$-axis, the onset of the first shear family of modes is 14% higher in velocity than the Rayleigh mode and the coupling to the Rayleigh mode is only 18% down from the standard cut. It is not likely that such a cut would be used though since one of the principal reasons for using quartz substrates is temperature stability, not plate mode freedom. If temperature stability were foregone as a requirement then materials with larger electromechanical coupling like LiNbO$_3$ would be considered. We have already seen from Figs. 2 and 8 that (xz) LiNbO$_3$ has adjacent plate modes farther out than those of standard ST-cut quartz, and the Rayleigh mode coupling from Table 1 is 4.4 times larger than the Rayleigh mode coupling of standard ST-cut quartz.

B. Lithium Niobate

Lithium Niobate is a material that has received considerable attention. Many people have reported [5], [8], [9], [11]-[14], [22]-[29], [32], [34], [37], [40], [42] on its bulk mode properties as they affect interdigital transducer excitations, and some of these investigators claim [26], [28], [42] to have found superior orientations for surface wave device work. The reasons for the interest in bulk mode properties of this particular crystal are probably threefold. First, it has become an industry standard being readily available commercially in large boules; second, next to a ceramic, it offers the highest electromechanical coupling known; and finally, it has notoriously bad bulk mode interference problems.
The \((yz)\)-cut of \(\text{LiNbO}_3\) is the most commonly used cut of the material. Plate mode distortion for this cut is extreme and can be seen in all experimental results using this cut unless efforts have been made to eliminate the modes. Mode coupling data for a \((yz)\) \(\text{LiNbO}_3\) plate are shown in Fig. 9. It can be seen that the first coupled mode is less than 4% higher in velocity than the Rayleigh mode and is only 30 dB down. The illustration shows that the train of modes continues in strength as the mode velocity increases. The study of these modes is what has occupied the interests of most investigators.

The first report [26] on another orientation of \(\text{LiNbO}_3\) which might have superior spurious mode properties made this claim for \(-41.5^\circ\) \((zx)\) \(\text{LiNbO}_3\). Unfortunately, the conclusions were based on experiments on large \(\text{LiNbO}_3\) boules in which they could only look 30 dB down from the Rayleigh mode. In experiments with adequate system sensitivity on substrates in the 0.040" thick range and frequencies in the 100-MHz range, one can easily ascertain that the \(-41.5^\circ\) \((zx)\) \(\text{LiNbO}_3\) cut has in-band spurious modes starting at about 30 dB.

Figure 10 shows the theoretical mode couplings for a plate of \(-41.5^\circ\) \((zx)\) \(\text{LiNbO}_3\). It will be noted that adjacent to the Rayleigh mode, the theoretical levels are at 47 dB; however, the density of modes is such that for most devices the aggregate spurious mode level is considerable higher than that. A graphic illustration of the limiting effect of the spurious modes in \(-41.5^\circ\) \((zx)\) \(\text{LiNbO}_3\) is given in Fig. 11 where in the upper trace resonator action is limited by the spurious mode levels in-band of the Rayleigh mode response. In the lower trace, the Rayleigh mode has been damped out revealing aggregate plate mode levels at \(-20\) dB.

Superior cuts of \(\text{LiNbO}_3\) have been predicted by examining inverse velocity plots for the properties described earlier. Rather than write a general search procedure to find the orientation that maximizes the plate mode properties of the crystal without regard for the other crystal parameters, we first eliminated most cuts on the basis of the properties. For example, unacceptable diffraction problems due to beam steering and highly non-parabolic velocity surfaces eliminates most cuts of \(\text{LiNbO}_3\). In fact, if diffraction needs to be considered in the filter design, then the standard cut, \((yz)\) \(\text{LiNbO}_3\), is a questionable choice for a substrate orientation.

One plane of \(\text{LiNbO}_3\) that we have considered and found useful is the \((yz)\) plane. Inverse velocity plots for the plane are shown in Fig. 12. The highly anisotropic nature of \(\text{LiNbO}_3\) is evident. As indicated in a previous section, this plane of \(\text{LiNbO}_3\) has one electrically uncoupled shear wave. That wave is labeled curve \(a\) in Fig. 12, and its polarization relative to the sagittal plane is
shown as perpendicular. Because the family of plate modes associated with curve $a$ is uncoupled from the transducer, the potential for using a $(zy)$ LiNbO$_3$ orientation is high. For an exactly $(zy)$ oriented plate and exactly $y$-oriented transducers, the first coupled plate mode would be $12.4\%$ higher in velocity than the Rayleigh mode. This is the largest known distance from the Rayleigh mode to the first coupled plate mode in LiNbO$_3$.

Unfortunately, other experimental parameters make this cut somewhat difficult to work with. The difficulty is that the uncoupling of branch $a$ is sensitive to the exact crystal cut, the exact transducer alignment along the $q$ axis, and the width of the transducer. The experimental errors in meeting these specifications tend to yield aggregate plate mode levels in the $-30$-dB to $-40$-dB range. Coupling calculations for the $(zy)$ LiNbO$_3$ orientation are shown in Figure 13. One can see by the enhancement of dots on the abscissa that many modes exist between the Rayleigh mode and the first coupled plate wave. A detailed treatment of the excitation of these modes which are uncoupled only for exactly $(zy)$ orientation is presented in Reference 51.

Returning to Fig. 12, one can see that there is a broad range of angles from about $30^\circ$ to $60^\circ$ above the $(010)$ axis, for propagation along any direction in that range, the bulk wave curves are about $10\%$ higher in velocity. Coupling plots for a plate with propagation at $46^\circ$ above the $(010)$ axis, a $46^\circ$ $(zyw)$ LiNbO$_3$ plate, are shown in Fig. 14. The expected gap with no modes above the Rayleigh mode is clearly predicted. Experimental results are shown for plates cut at $45^\circ$ and $42^\circ$ in the inset in Fig. 14 and in Fig. 15, respectively. The quiescent gap is apparent in the Fig. 14 inset photograph and was brought out in the Fig. 15 photograph by taking the upper trace with broad-band transducers and the crystal unperturbed and then repeating the process for the lower trace with the Rayleigh mode absorbed from the crystal.

Additional properties of cuts taken from the $30^\circ$ to $60^\circ$ range defined above are shown in Table I. One property of interest is the Rayleigh mode coupling. This is shown in Fig. 16 where the range with distant plate modes is shown between dashed lines. It will be noted that the coupling goes from a value of about 8 times that of standard ST-cut quartz to a value equal to that of $(yz)$ LiNbO$_3$. Thus a filter designer can choose the coupling appropriate to his filter requirements. It will also be seen from the data in Table I that the velocity surface for the $46^\circ$ $(zyw)$ LiNbO$_3$ cut is more nearly parabolic than the standard $(yz)$ LiNbO$_3$ cut.

One other cut of LiNbO$_3$ for which we have calculated the plate mode spectrum is $(yz)$ LiNbO$_3$. This orientation for surface wave propagation commonly arises in RAC devices on $(yz)$ LiNbO$_3$. The Rayleigh wave, when reflected off the chevrons along the $z$ direction, propagates in the $x$ direction a short distance before encountering the second array of chevrons and becoming $z$-propagating...
again. The mode couplings for this cut are shown in Fig. 17. At nearly 22% above the Rayleigh velocity, the mode couplings reach their peak which is considerably in excess of the coupling to the Rayleigh wave.

Figures 8 and 17 can be used to visualize the relation of pseudosurface waves to plate waves. Pseudosurface wave propagation along the $x$-axis of a LiNbO$_3$ half space has been analyzed \cite{22}, \cite{23} as a function of substrate normal. They found that for $(g_x)$ LiNbO$_3$, there was a pseudosurface wave at about 24% higher velocity than the Rayleigh mode with approximately 16 times the Rayleigh mode coupling. For the $(zr)$ LiNbO$_3$ cut, they found the pseudosurface wave at about 29% higher velocity and with about 11 times the Rayleigh coupling. In Figs. 8 and 17, we find that the plate mode coupling peaks at approximately the relative velocities corresponding to the pseudosurface wave velocity of the half space but the coupling to any one of the many modes is less than that of the single pseudosurface wave.

C. Bi$_2$GeO$_6$ and Bi$_2$SiO$_6$

Finally, two other materials that have had some degree of commercial interest are bismuth germanium oxide (BGO) and bismuth silicon oxide (BSO). BGO has been used for several years now but has had resistance to general acceptance because commonly employed metals do not form lasting contacts on the surface of the crystal. BSO is a more recent addition to the list of crystals examined for commercial surface wave device use. Mullard in England has championed the use of this material and claims to be using it for their TV IF filters. One of their investigators recently showed \cite{28} that the 45° $(zsl)$ cut of BSO can be made relatively free of spurious mode interference by proper treatment of the bottom surface of the substrate.

BGO and BSO have virtually identical piezoelectric stress constants; their densities are within about 0.1% of each other; and the dielectric constants are within 10% of each other. Consequently, the acoustic behavior of the two materials is very similar. If their inverse velocity curves are laid over top of one another, they are nearly coincident. Thus theoretical calculations carried out for one material are a good indicator of the acoustic behavior of the other material. Figures 18 and 19 are illustrations of calculations for BGO and experiments on BSO. Both theory and experiment were carried out for the most commonly used orientation of BGO, namely 45° $(zst)$. Strong spurious mode levels 37 dB below the Rayleigh mode response are evident in the inset in Fig. 19. They are lower than the theoretical mode levels shown in Fig. 19. However, the calculation was for a specific plate thickness $t/\lambda = 60/2\pi$ and also indicates the levels that would occur if the transducer spectrum were infinitesimally narrow and if there were no losses at the bottom surface of the crystal. None of these conditions were met in obtaining the experimental results.

Shown in Fig. 20 are theoretical calculations of the plate mode levels of BSO for the orientation reported by Mullard \cite{28} to be superior, the 45° $(zst)$ cut. The cou-
The higher modes of these cuts interact strongly with the bottom surface and can be reduced significantly by the techniques of Ref. 39. Finally, substrates cut with these orientations provide a range of coupling coefficients and predictable diffraction properties.

APPENDIX A

DISCUSSION OF TABLE I

A number of parameters need to be taken into consideration in choosing a substrate for an acoustic surface wave filter. Three factors which need to be considered, in addition to spurious mode interference, are: the effect of diffraction on the Rayleigh mode, thermal sensitivity to time delay variation of the Rayleigh mode, and electromechanical coupling to the Rayleigh mode. Data for these three properties are given in Table I. The data are arranged in the order of decreasing Rayleigh wave velocity. Only two parameters are missing from the table; the coefficients of time delay for BSO and BGO have not been published. The other time delay coefficients were taken from Ref. 46. Exception for the temperature coefficients of time delay, the rest of the data in Table I were computed by the author using published material constants [47]-[50].

A parameter of extreme importance which is not included in Table I is the beam steering or acoustic walk-off angle of the Rayleigh mode. This is the angle between the phase velocity and the group velocity of the wave. For all of the cuts shown in the table, this angle is zero because we have restricted ourselves to the study of only those cuts which have no beam steering. Including this effect in the design of state-of-the-art band-pass filters requires not only extremely sophisticated theoretical modeling capabilities but, for narrow beamwidth devices, exacting fabrication accuracy.

Temperature Coefficient of Time Delay

The temperature coefficient of time delay acceptable for a given filter design will, of course, be determined by the system application. If the system can AFC (automatic frequency control) to adapt to temperature changes, then a crystal that is moderately sensitive to temperature but which offers high coupling, like (yz) LiNbO₃, might be used. On the other hand, if the system is nonadaptive and a thermally varying environment is anticipated, then the most stable crystal orientation, 47.25° (zr1) quartz is indicated even though the Q of the transducer impedance for that cut is very large. Table I shows that except for 47.25° (zr1) quartz, all of the crystal cuts have coefficients of time delay in the 70-90 parts per million range. Consequently, on the basis of thermal properties, the substrates break down into two categories, insensitive (47.25° (zr1) quartz) and sensitive (all the others). In the sensitive group one substrate is about as good as another.
Rayleigh Mode Diffraction

The Rayleigh mode experiences diffraction in traveling from the input transducers to the output transducer because the transducers are of finite width. Diffraction effects vary with substrate orientation because the crystals are anisotropic and have different surface wave velocities in different directions. Diffraction would occur even on an isotropic substrate because of finite transducer width. However, on the anisotropic crystals, listed in Table I, the effect is either accelerated or retarded according to the velocity variation with propagation direction.

If $\theta$ is an angle in the plane of the substrate measured away from the boule axis of the substrate, then the velocity of a Rayleigh wave on the crystal is given by the equation on the bottom of Table I where the coefficients are those in the table. The equation is valid for small angles (less than $5^\circ$). One can see that the coefficients are small and the velocity variation is correspondingly slight. However, this small charge is responsible for controlling the diffraction of the wave.

One approximation widely used in describing diffraction results is the so-called "parabolic approximation" where $V(\theta)$ is approximated by $V_0 [1 + a \theta^2]$ and the other terms are ignored. The coefficients of Table I show, qualitatively, the error in making this approximation. For example, they show that it is a more valid approximation for $46^\circ$ (yzc) LiNbO$_3$ where $a_2/a_1 \cong -1896$ then it is for (yz) LiNbO$_3$ where $a_2/a_1 \cong -99$.

If the Rayleigh mode velocity decreases with increasing angle, the crystal cut is a focusing direction and diffraction effects are retarded. Conversely, if the Rayleigh mode velocity increases with increasing angle, diffraction effects are accelerated. Those cuts in Table I with $a_2 < 0$ are focusing.

Mode-Free Range

For narrow-band IF filtering, an important consideration is the width of the gap above the Rayleigh mode to the first coupled plate mode. This mode-free range, MFR, is indicated in Table I as a percentage of the Rayleigh mode velocity.

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REFERENCES

Elastic Constants and Thermal Expansion of Berlinite

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Abstract—The six single crystal elastic constants of α-berlinite, α-AlPO₄, have been measured between 80°K and 298°K by the ultrasonic pulse superposition method, and the thermal expansion behavior has been determined from 293°K to 950°K by the X-ray powder diffraction method through the α→β transition at about 857°K.

All results are very similar to those for α-quartz. With the exception of c₄₄ and c₅₅, all elastic constants decrease with increasing temperature, but c₆₆ and | c₆₆ | increase monotonically. As a result, temperature compensated cuts with zero temperature coefficient of the resonance frequency at 25°C are found, with orientations similar to those for the AT and BT cuts in α-quartz, but with a larger electromechanical coupling factor.

I. INTRODUCTION

BERLINITE, AlPO₄, is of interest because of the similarity of its crystal structure and properties with quartz, SiO₂. The berlinite structure is derived from the quartz structure by replacing the Si ions with Al and P ions, which results in a doubling of the unit cell along the c axis [1]. The associated size reduction of the first Brillouin zone leads to a doubling of the number of IR and Raman active modes at the zone center, with the extra modes corresponding to the critical point A at the zone boundary in quartz [2]. Because of the small mass differences between the cations, seven of the eight Raman frequencies in AlPO₄ are very nearly the same as the corresponding frequencies in SiO₂, but the frequency of the temperature dependent soft mode associated with the α→β phase transition is higher [2] in AlPO₄ than in SiO₂. The α→β transition itself, however, occurs at almost the same temperature [3], [4] (584°C in AlPO₄ and 573°C in SiO₂) and has the same pressure coefficient for both materials [3], [4]. The room-temperature values of the elastic constants exhibit considerable differences, with the on-diagonal elastic moduli differing by up to a factor of 2.5 and the off-diagonal modulus c₆₆ differing by a factor of six [5]–[7]. The temperature coefficients of the elastic constants of berlinite are not known. It has been reported [7], however, that the resonance frequency for the Y-cut thickness-shear mode shows a maximum at −40°C. This behavior is in contrast to that of quartz, for which several “temperature compensated cuts” exist at room tempera-